PREDICTING LATERAL DEFLECTION AND FUNDAMENTAL NATURAL PERIOD OF MULTI-STOREY WOOD FRAME BUILDINGS

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ABSTRACT: The Canadian timber design code provides a model to calculate lateral deflection of a wood stud shear wall. Modification of the model is necessary if it is to be used for predicting deflection of buildings two storeys or taller. Moreover, the design of a multi-storey building to resist wind and seismic loads requires that the fundamental period of the building be known. Currently, the National Building Code of Canada (NBCC) provides an empirical model to predict fundamental natural period of a building based on height only. This equation was derived from an analysis of dynamic responses of a number of steel and concrete buildings during seismic events in California, USA. As a result, its applicability to wood frame buildings in Canada is questionable. Therefore, there is a need to develop design tools to predict lateral deflection and fundamental natural period of multi-storey wood frame buildings. This need has become more pressing with the initiatives in some Canadian provinces raise the height limit for wood frame buildings from the traditional four-storey. By applying the compatibility condition for deflection at each storey level, a mechanic-based model has been developed to estimate lateral deflections of multi-storey light wood frame buildings. Based on an analysis of a six-storey light wood frame building, a comparison of the predicted deflection using the mechanics-based model with the predictions from frame analysis and finite element programs has verified the capability of the proposed mechanics model to estimate lateral drift of the building. This work has also provided preliminary evidence that it is feasible to use a stick frame modelling approach to analyze complex 3-D light wood frame structures. When it comes to prediction of fundamental natural period of the building, three methods were investigated: stick frame, finite element and simplified one degree-of-freedom cantilever models. It was found that the predicted values for the fundamental natural period using the three methods differ significantly. Interestingly the finite element model produced natural period value that was close to that predicted by the NBCC empirical formula. It was concluded that this discrepancy for natural period prediction could be related to the differences in accounting for the modal mass in these three approaches and further work is required to investigate this issue further.

KEYWORDS: Multi-storey wood frame buildings, lateral load, base shear design, drift, fundamental mode, finite element model.

1 INTRODUCTION

There are number of uncertainties in lateral designs of wood light-frame buildings (WLFB). Unlike other construction materials, wood construction is segmental and rarely designed with rigid connections. As such, lateral load resisting systems in wood construction are typically shear walls or braced frame systems. These uncertainties include estimating fundamental natural period, assuming rigid versus flexible diaphragms, predicting shear wall deflection (drift) at each storey, and conducting dynamic analysis.

Fundamental period: The National Building Code of Canada (NBCC) suggests that buildings utilizing shear wall systems as Lateral Force Resisting System (LFRS) may use the formula 0.055(hₙ)⁰.⁷⁵ to estimate the fundamental period of a building, where hₙ is the total building height in meters [1]. Based on this formula, the period of any building with shear wall system is proportional to the height of the building. For example, to design a building with a maximum period of 0.5 sec such that static equivalent method in seismic design is applicable, the height of a building with any material should not be taller than 21.5 m. The formula in estimating the fundamental period of buildings with shear wall system was developed based on buildings other than wood construction. There appears to be no evidence of formal studies yet done for the fundamental period of multi-storey WLFB. Intuitively, WLFB is more flexible than concrete, steel or masonry construction, if they are not designed to meet the same deflection limit. As such, a WLFB should deflect more
than buildings constructed with other materials under the same external influence. With a larger deflection for the same building height, a WLFB is expected to have a larger fundamental period than buildings with other material.

Flexible or rigid diaphragm: Whether a floor or roof plate behaves in a flexible or rigid manner under lateral loads has been debated for years with no satisfactory conclusions. Structural Engineering association of California (SEAOC) recommends that a building’s floor and roof plates be assumed to be flexible in the initial calculations to distribute the loads for the design of shear walls [2]. From the designed walls and the distributed loads estimated from flexible diaphragms, the deflections of the shear walls can then be estimated. With the known deflections, stiffnesses of the walls are estimated. The floor and roof plates are then assumed to be rigid and lateral forces are again redistributed using the estimated wall stiffnesses. The shear walls of the building would be redesigned using the envelope forces generated from both methods.

Shear wall deflection: The shear wall deflection equation as provided by the Wood Design Manual is consistent with deflection equation developed at other countries [3]. However, a close examination of the equation has revealed that the equation only describes the deflection of a single level shear wall, taking no account of the simultaneous loading effects of multi-levels and the cumulative effects of individual storey rotation and deflection.

Dynamic analysis: With wood framed buildings being segmental in their construction, along with incomplete deflection estimation of shear walls, modeling light wood frame construction for an entire building for dynamic analysis has rarely been done. Thus, dynamic analysis in designing WLFB is generally avoided while the equivalent static analysis method has been traditionally used.

2 OBJECTIVE

The main objective of this study is to propose modifications to lateral designs for WLFB with wood panel shear wall systems.

With the current NBCC lateral design requirements, particularly in seismic designs, it is necessary to determine the overall behaviour of a structure. With the exception of buildings in low seismicity area, regular structures of less than 60m in height with fundamental lateral period less than 2 sec, or irregular structures other than torsional sensitive structure less than 20m in height in building with fundamental lateral period less than 0.5 sec, NBCC states that dynamic analysis procedure shall be used as the basis of design for all buildings [1]. Therefore, characteristics of a building such as its overall stiffness, deflection, torsional sensitivity and fundamental period are critical parameters in determining the structural behaviour in a seismic event.

The current edition of Canadian timber design standard, CSA O86 Engineering Design in Wood, provides a method of calculating lateral deflection of single level shear walls [4]. The standard is silent on the need to include influence of multi-level loading and cumulative rotation and deflection. Furthermore, it is customary to assume that all wood floor diaphragms are flexible to avoid investigation of torsional effects in wood buildings.

These simplifications may be adequate for low-rise structures with 4 storeys or less. The overall building’s structural behaviour, however, cannot be captured by these simplified methods. For example, the fundamental period of low-rise WLFB is currently estimated using an empirical formula that was derived in the 1960’s from studies of concrete and steel structures. With the inclusion of taller six level wood framed structures, it is imperative that the structural behaviour under lateral load conditions for wood framed buildings be better handled.

It is the intention of this study to provide a more comprehensive approach to better estimate building deflections and stiffnesses under lateral loads using engineering mechanics concepts. By considering all wall stiffnesses of a building three dimensionally, a more accurate estimate of the fundamental period of the building based on multi-degree of freedom can be found using a commercial structural analysis program as a tool. With this approach, the torsional effects of the overall building can be captured while the base shear of a building can be estimated more accurately.

3 SHEAR WALL DEFLECTION

To predict total lateral deflection (drift) of a multi-storey WLFB under lateral load, shear wall deflections need to be determined at each storey level. The current accepted shear wall deflection formula in CSAO86-01 [4] is:

$$\Delta_{sr} = \frac{2v_{sr}H_{sr}^3}{3EAL_w} + \frac{v_{sr}H_{sr}}{B_v} + 0.0025H_{sr}e_n + \frac{H_{sr}}{L_w} \Delta_{dr}$$

(1)

where the first term is the bending deflection due to point load applied perpendicularly to the tip of a cantilever vertical beam and is derived from the term:

$$\Delta = \frac{PH^3}{3EI}$$

(2)

where:
P is a point load at the top of the wall,
Hw is the height of the wall,
I is the moment of inertia and can be written as $\frac{1}{2}AL_w^2$,
with “A” being the area of the end vertical stud of the shear wall and “L_w” being the length of the shear wall, and
E is Young’s modulus.

The term is applicable for a single point load applied to top of a single level shear wall with a height of Hw. If the same wall at multiple levels is simultaneously experiencing multiple point loads at various locations, the resulting internal moment and shear forces at each storey must be accounted for. This also implies that
compatibility condition for deflection at each storey must be fulfilled. The following is the derivation of modified terms of equation (1).

**Derivation of the modified 1st term:** The following graphical diagram (Fig 1) and equations are used to estimate the modified bending deflection of a multi-level loaded shear wall at various levels of the wall, using six storey WLFB as an example for calculation.

Based on the lateral loads applied as shown in Fig 1, the shear at each storey is

\[
V_0 = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 \\
V_1 = V_0 - P_1 \\
V_2 = V_1 - P_2 \\
V_3 = V_2 - P_3 \\
V_4 = V_3 - P_4 \\
V_5 = V_4 - P_5 \\
V_6 = V_5 - P_6
\]  

and the associated moment is:

\[
M_0 = V_0 H_1 + V_1 H_2 + V_2 H_3 + V_3 H_4 + V_4 H_5 + V_5 H_6 \\
M_1 = M_0 - V_0 H_1 \\
M_2 = M_1 - V_1 H_2 \\
M_3 = M_2 - V_2 H_3 \\
M_4 = M_3 - V_3 H_4 \\
M_5 = M_4 - V_4 H_5 \\
M_6 = M_5 - V_5 H_6
\]

**Fig. 1: Shear and moment**

Fig. 2 shows the deflected shape of the structure, based on the shear and moment developed under the applied lateral loads. The rotation at each storey can be calculated as follow:

\[
\theta_0 = 0 \\
\theta_1 = \left[\left( (M_0 H_1) - \left( V_0 H_1^2 / 2 \right) \right) / (EI_1) \right] + \theta_0 \\
\theta_2 = \left[\left( (M_1 H_2) - \left( V_1 H_2^2 / 2 \right) \right) / (EI_2) \right] + \theta_1 \\
\theta_3 = \left[\left( (M_2 H_3) - \left( V_2 H_3^2 / 2 \right) \right) / (EI_3) \right] + \theta_2 \\
\theta_4 = \left[\left( (M_3 H_4) - \left( V_3 H_4^2 / 2 \right) \right) / (EI_4) \right] + \theta_3 \\
\theta_5 = \left[\left( (M_4 H_5) - \left( V_4 H_5^2 / 2 \right) \right) / (EI_5) \right] + \theta_4 \\
\theta_6 = \left[\left( (M_5 H_6) - \left( V_5 H_6^2 / 2 \right) \right) / (EI_6) \right] + \theta_5
\]

and the associated inter-storey drift is:

\[
\Delta_{B_0} = 0 \\
\Delta_{B_1} = \left[\left( (M_0 H_1^2) / 2 - \left( V_0 H_1^3 / 6 \right) \right) / (EI_1) \right] + \Delta_{B_0} \\
\Delta_{B_2} = \left[\left( (M_1 H_2^2) / 2 - \left( V_1 H_2^3 / 6 \right) \right) / (EI_2) \right] + \Delta_{B_1} \\
\Delta_{B_3} = \left[\left( (M_2 H_3^2) / 2 - \left( V_2 H_3^3 / 6 \right) \right) / (EI_3) \right] + \Delta_{B_2} \\
\Delta_{B_4} = \left[\left( (M_3 H_4^2) / 2 - \left( V_3 H_4^3 / 6 \right) \right) / (EI_4) \right] + \Delta_{B_3} \\
\Delta_{B_5} = \left[\left( (M_4 H_5^2) / 2 - \left( V_4 H_5^3 / 6 \right) \right) / (EI_5) \right] + \Delta_{B_4} \\
\Delta_{B_6} = \left[\left( (M_5 H_6^2) / 2 - \left( V_5 H_6^3 / 6 \right) \right) / (EI_6) \right] + \Delta_{B_5}
\]

**Fig. 2: Rotation and inter-storey drift**

**Derivation of the modified 2nd term:** The second term of equation (1) describes the wood panel shear deformation and is derived from the term:

\[
\Delta = \frac{PH}{A_y G}
\]

where \(G\) is the shear modulus of the wood panel, and \(A_y\) is the shear area of the shear wall. The code shear deflection is obtained by substituting \(v_{sr}\) equal to \(P/L_w\) and \(B_v\) equal to \(G^*t\) where \(t\) is the thickness of the sheathing panel.
Using schematic shear deformation shown in Fig 3 for the calculation of shear deflection for each floor, the following equations summarize shear deflection at each storey.

\[
\Delta V_1 = \frac{(V_0 \cdot H_1)}{(L_w \cdot t \cdot G)} \\
\Delta V_2 = \frac{(V_1 \cdot H_2)}{(L_w \cdot t \cdot G)} \\
\Delta V_3 = \frac{(V_2 \cdot H_3)}{(L_w \cdot t \cdot G)} \\
\Delta V_4 = \frac{(V_3 \cdot H_4)}{(L_w \cdot t \cdot G)} \\
\Delta V_5 = \frac{(V_4 \cdot H_5)}{(L_w \cdot t \cdot G)} \\
\Delta V_6 = \frac{(V_5 \cdot H_6)}{(L_w \cdot t \cdot G)} \quad (7)
\]

Fig. 4: Shear deformation due to nail slip

**Derivation of the modified 3rd term:** Fig 4 shows the schematic deflected shape due to nail slippage. The amount of slippage of a nail at a specified load level is derived from experiment and the nail slippage given (Table 1) is taken from the Wood Design Manual [3]. The 3rd term of the shear wall deflection equation given in the average load level exerted on each nail along each floor of the shear wall and cross reference with the table to obtain a value for nail slippage. The relationship of the nail slippage and load exerted on the nails is non-linear and iteration of the load may be required to refine calculations. For calculation of nail slip deflection of each floor of the shear wall, the following equations summarize the nail slip calculations:

\[
\Delta NS_1 = 0.0025 \cdot H_1 \cdot e_{n1} \\
\Delta NS_2 = 0.0025 \cdot H_2 \cdot e_{n2} \\
\Delta NS_3 = 0.0025 \cdot H_3 \cdot e_{n3} \\
\Delta NS_4 = 0.0025 \cdot H_4 \cdot e_{n4} \\
\Delta NS_5 = 0.0025 \cdot H_5 \cdot e_{n5} \\
\Delta NS_6 = 0.0025 \cdot H_6 \cdot e_{n6} \quad (8)
\]

where \(e_{n1} \), \(e_{n2} \), \(e_{n3} \), \(e_{n4} \), \(e_{n5} \) and \(e_{n6} \) are taken from Table 1 (which in principle is a function of “N/nail” and nail length).

Table 1: Nail load-slip data

<table>
<thead>
<tr>
<th>N/nail</th>
<th>en</th>
<th>en - 2.5“nail</th>
<th>en - 3”nail</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.29</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>0.46</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.64</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>0.88</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>1.21</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>1.7</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>2.33</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td>1.95</td>
<td></td>
</tr>
</tbody>
</table>

and “N/nail” is function of applied lateral load, shear wall load, and nail spacing at each storey, as follow:

N/nail = \(V_0 / (L_w / Nail \ Space \ at \ level \ H_1)\)
N/nail = \(V_1 / (L_w / Nail \ Space \ at \ level \ H_2)\)
N/nail = \(V_2 / (L_w / Nail \ Space \ at \ level \ H_3)\)
N/nail = \(V_3 / (L_w / Nail \ Space \ at \ level \ H_4)\)
N/nail = \(V_4 / (L_w / Nail \ Space \ at \ level \ H_5)\)
N/nail = \(V_5 / (L_w / Nail \ Space \ at \ level \ H_6)\)

where \(V_i\) is the shear force at storey \(i\).

Fig. 5: Shear deformation due to nail slip

**Derivation of the modified 4th term:** The 4th term of Equation (1) is the lateral deflection due to anchorage deformation. In that term, \(\Delta ar\) denotes the estimated vertical deformation due to the hold-down device’s elongation or crushing of the chord members at one level only. As seen in Fig 5, each level’s horizontal deflection is a result of the cumulative rotational effect of the lower level’s vertical deformation in addition to the deformation of each corresponding level. As such, for a cumulative multi-level shear wall effect, the following is a summary of the equations used to estimate each storey’s horizontal deflection taking into account the effects of the lower levels.
\[ \Delta h_{d1} = H_1 \cdot \frac{\Delta a_{r1}}{L_w} \]
\[ \Delta h_{d2} = H_2 \cdot \frac{(\Delta a_{r1} + \Delta a_{r2})}{L_w} \]
\[ \Delta h_{d3} = H_3 \cdot \frac{(\Delta a_{r1} + \Delta a_{r2} + \Delta a_{r3})}{L_w} \]
\[ \Delta h_{d4} = H_4 \cdot \frac{(\Delta a_{r1} + \Delta a_{r2} + \Delta a_{r3} + \Delta a_{r4})}{L_w} \]
\[ \Delta h_{d5} = H_5 \cdot \frac{(\Delta a_{r1} + \Delta a_{r2} + \Delta a_{r3} + \Delta a_{r4} + \Delta a_{r5})}{L_w} \]
\[ \Delta h_{d6} = H_6 \cdot \frac{(\Delta a_{r1} + \Delta a_{r2} + \Delta a_{r3} + \Delta a_{r4} + \Delta a_{r5} + \Delta a_{r6})}{L} \]
\(__9__) 

4 SIX-STOREY WLFB ANALYSIS

With the above shear wall deflection formulas as defined (equations 6, 7, 8 & 9), it is suggested that a building’s lateral force be distributed by assuming that the floor and roof plates are flexible. The shear forces distributed to each shear wall line are based on the corresponding tributary area. The shear wall design is detailed according to the loading as determined and the deflections are calculated by taking the entire building into account.

For the purpose of this study, an example of a six-storey building is used to demonstrate the approach (see Fig 6). The sample building is 9.14m by 9.14m in plan with each level having 3m floor-to-floor height. There are 5 shear walls with 2 walls in the left-right direction and 3 walls in the up-down direction, each with 15 feet (4.57m) length.

Table 2 shows the loading data for the roof and floors. The total building weight is 792.51 kN. Using the NBCC Code formula [1], the fundamental period of the building is 0.437 seconds derived from the 18 m tall building. Thus, using Code formula with zoning applicable to Vancouver area (heavy earthquake region), the base shear of the building is 107.12 kN with combined R_dR_o of 5.1, where R_dR_o is intended to account for ductility and over-strength related factors for wood structure. Since the building is regular and has a period of less than 0.5 seconds, equivalent static force method is applicable. The base shear is distributed inversely proportional to the height of the building and shear forces are distributed to the walls by assuming the diaphragms are flexible. Table 3 shows a summary of calculation result for the lateral load distribution at each floor.

Table 3: Lateral load distribution

<table>
<thead>
<tr>
<th>Level</th>
<th>Lateral Load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof level</td>
<td>23.52 kN</td>
</tr>
<tr>
<td>6th floor</td>
<td>27.87 kN</td>
</tr>
<tr>
<td>5th floor</td>
<td>22.29 kN</td>
</tr>
<tr>
<td>4th floor</td>
<td>16.72 kN</td>
</tr>
<tr>
<td>3rd floor</td>
<td>11.15 kN</td>
</tr>
<tr>
<td>2nd floor</td>
<td>5.57 kN</td>
</tr>
<tr>
<td>Base shear</td>
<td>107.12 kN</td>
</tr>
</tbody>
</table>

To calculate shear wall deflection, these lateral loads need to be distributed to the shear walls using flexible diaphragm assumption (tributary area method). To do this, wall material properties are needed. The shear wall used has a 2x6 framing system with Douglas-Fir #2 lumber having E=11,000 MPa. The wall sheathing was 12 mm thick plywood with G*t=11,000 N/mm. Nails with 2.5 inches (63.4 mm) length are used to connect wall sheathing-to-framing for 4th, 5th, 6th, and roof levels, while 3 inches (76.2 mm) nails were for 2nd and 3rd levels. Edge nail spacing is 100 mm at the roof level, 75 mm at the 6th level, and 50 mm at the 5th, 4th, 3rd and 2nd levels. Steel hold-downs (E=200 GPa) are used at the wall ends, with 25.4 mm-diameter bolts applied at 5th to roof levels, and 32 mm-diameter bolts for 2nd to 4th levels. In this study, gypsum wall board was not included in the analysis.

The deflection of each wall is calculated according to the modified deflection calculations (equations 6 to 9). Also, the traditional method of calculating shear wall deflection (equation 1) is included as a comparison. Table 4 summarizes the drift deflection per level for the walls using both methods. Walls A and B have the same results because of symmetry and likewise for Walls 1 and 3. The results show that the deflections of the shear walls increase significantly if multi-level effects are taken into account.
From the calculated total drift at the roof level (71.60 mm), 13.68mm was from the bending contribution, 13.36mm from the shear panel deflection, 22.98mm from the nail slippage, and 21.58mm from the hold-down anchorage deformation.

Table 4: Drift comparison

<table>
<thead>
<tr>
<th>Wall A &amp; B</th>
<th>Wall 1 &amp; 3</th>
<th>Wall 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod* Code</td>
<td>Mod* Code</td>
<td>Mod* Code</td>
</tr>
<tr>
<td>Roof</td>
<td>10.67</td>
<td>2.71</td>
</tr>
<tr>
<td>6th</td>
<td>13.12</td>
<td>5.74</td>
</tr>
<tr>
<td>5th</td>
<td>12.82</td>
<td>6.55</td>
</tr>
<tr>
<td>4th</td>
<td>13.23</td>
<td>8.34</td>
</tr>
<tr>
<td>3rd</td>
<td>11.66</td>
<td>8.68</td>
</tr>
<tr>
<td>2nd</td>
<td>10.10</td>
<td>9.63</td>
</tr>
<tr>
<td>Total</td>
<td>71.60</td>
<td>41.64</td>
</tr>
</tbody>
</table>

Note: *Modified drift calculation

5 STICK FRAME MODEL

With the characteristics of a multi-level shear wall determined by taking the entire building into account, it is possible to model the shear wall as a stick model with the same stiffness and strength characteristics as a wood framed shear wall for computer modeling. There are three elements of the shear wall that the stick model has to capture so that a commercial frame analysis program can be used.

(1) **Moment of inertia:** For all commercially available programs, if the input of the shear wall cross section includes the web thickness, the program would account for the web’s moment of inertia as well. In the bending deflection equation, the sheathing’s moment of inertia is ignored. As such, the moment of inertia in the computer modeling should be adjusted to reflect the reduction in the moment of inertia.

(2) **Equivalent shear area:** The 2nd and 3rd term of the deflection equation include the panel shear deformation and the nail slip deformation. For both terms, the deflections are in the form of shear. However, the nail slip term is nonlinear. Yet, an approximation is proposed to use the value of the deflection generated from the loads of each floor. The nail slip deflection is added to the panel shear deformation and combined to be called the total shear deflection. Based on this combined shear deflection, an equivalent shear area, $A_v$, can be determined for the shear wall of that level. The stick model representing the shear for each level would have an equivalent shear area, $A_v$, to include nail slip deformation.

(3) **Rotational Spring:** The fourth term of the deflection equation accounts for the uplifting or crushing of the shear wall due to overturning moments. This effect of the shear wall can be modeled with a rotational spring at each floor. With the vertical deformation, $\Delta_\alpha$, estimated and the wall length, $L_w$, known, the angle of rotation, $\alpha$, can be estimated. With the net overturning moment calculated and divided by the angle of rotation, the stiffness of the rotational spring can be established.

With the above approximations, a shear wall can be modeled as a stick frame member with the same characteristics as the shear wall (Fig 7a).

In this study a commercial structural analysis software S-Frame was used to perform this simplified analysis [5]. Comparing the result of calculated deflection of the shear wall A shown in Table 4 with the output from S-Frame, it is found that the deflections are similar. Fig 7b shows the deflected shape of the stick-frame model of Wall A.

With all the walls modeled as members in the above manner, the sample building can be represented in a 3-dimensional model comprising of stick members in both the x and y direction. Furthermore, in the modeling of the entire building, the floor and roof diaphragms are assumed to be rigid in order to generate a lump mass and mobilize all members together. Fig 8 shows the computer model for the six-storey WLFB with the 5 shear walls incorporated into the stick model.

With the complete building modeled and the stiffness of the shear walls all accounted for, the fundamental periods of the building can be estimated using the computer program. Fig 9 illustrates the outputs of the first 3 mode shapes of the building. The first two modes account for close to 80% of the mass with the period of
the first 2 mode being 1.14 second and 0.96 second, respectively. The calculated period for this building using the method described here differs substantially from the building period estimated from the Building Code formula [1].

![Mode shapes and period](image)

(a) Mode 1, \( T_a = 1.14 \) s  
(b) Mode 2, \( T_a = 0.96 \) s  
(c) Mode 3, \( T_a = 0.37 \) s

Fig. 9: Mode shapes and period

According NBCC (Article 4.1.8.11.3(d)(iii)), the fundamental period calculated by the program cannot be greater than two times the period calculated by the formula \( 0.05 \times (h_0)^{0.75} \) [1]. The fundamental period \( T_a \) of the building calculated for this building using the NBCC formula is 0.437 seconds. As such, the maximum period that can be used for calculating base shear is 0.874 seconds. Accordingly, the base shear design for the building using the procedure described in NBCC (Article 4.1.8.11.2) with \( T_a \) equal to 0.874 seconds would result in a base shear of 69.7 kN rather than the originally estimated base shear of 107 kN.

With the above approach, the building with wood frame shear walls can be modeled as a stick model and a linear dynamic analysis can be performed. Dynamic analysis procedure can then be followed. For example, the base shear value, \( V_d \), can be obtained by dividing \( R_d \) into the base shear of the building obtained from the dynamic analysis (Art. 4.1.8.12(5)). In this case, in the x-direction, \( V_d \) is equal to 50.39 kN and in the y-direction, \( V_d \) is equal to 57.83 kN However, the Building Code has a number of restrictions with results from dynamic analysis as a safe guard to uncertainties. It is necessary to check for the minimum allowable base shear of the building. As an example, NBCC (Art. 4.1.8.12(6)) states that the base shear \( V_d \) shall not be less than 80% of the base shear calculated in accordance with Art. 4.1.8.11. In this case, the base shear calculated in accordance with Art. 4.1.8.11 is 69.7 kN as shown previously using the maximum allowed period of 2 times the period calculated from the Code formula. 80% of 69.7 kN is 55.76 kN. As such, while the y-direction base shear, \( V_d \), satisfies Art. 4.1.8.11, x-direction base shear, \( V_d \), should be 55.76 kN instead of the calculated value. In this discussion paper, the effects of accidental torsional moments have not been included for simplicity in the discussion. However, with the rigid diaphragm assumptions in the computer modeling, it is relatively easy to incorporate the torsional effect.

6 DETAILED 3-D FINITE ELEMENT MODEL

Another level of 3-D modelling for the 6-storey WLFB studied here was conducted using finite element software SAP2000 [6]. This time the modelling was conducted in detail incorporating actual wall panel arrangements, nailing patterns and their associated stiffness properties, as well as interconnection between shear walls and floor. Using current computer technology, building a detailed 3-D model of a multi-storey building can be achieved with relative easy. However, modelling at this component level is still not a common practice in present engineering design practice.

![Drift comparison](image)

Fig. 10: Drift comparison (*calculated drift in Table 4)

The modelling technique developed here followed the procedure outlined in a WCTE 2010 companion paper [7]. All wood material and connection properties used were consistent with the data described in the previous section, with some nail stiffness properties taken from the literature [e.g. 8]. Floors and roof were modeled as rigid, but with automatic masses assigned via inputting wood panel and frame densities and associated cross sectional properties. Rigid behaviour in this model was simulated using infinite (high value) stiffness assigned to the floor framing. The boundary conditions between the super-structure and the ground level were modeled as hinges.

The lateral loads obtained from the Code based calculation (Table 3) were applied uniformly to the floor and roof diaphragms. Fig 10 shows the comparison...
between the predicted drift using the 3-D FE model and the modified CSA O86 calculation procedure (Table 4). It can be seen that close agreement was obtained. This also implies that the predicted drifts are almost similar to those predicted using the stick frame model as described in section 7.

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Fig 11 shows the first three modes and associated periods of the detail 3-D FE model. It can be seen that the first-mode period is 0.459 s, which is similar to the fundamental period calculated according to NBCC (0.437 s) [1]. This predicted value is significantly different from that obtained using the stick frame model.

7 DISCUSSION

Major discrepancies in term of predicting fundamental period were obtained between the stick frame and detail 3-D FE model. Since the calculated deflections were similar the discrepancy is likely due to the differences in the way the mass influence was accounted for in different modelling approaches. One of the sources of discrepancy could be due to modeling assumption about concrete topping and other non-structural materials in the diaphragms that result in a higher mass, and therefore higher structural period. However, recent test result of a six-storey test building loaded under a real earthquake excitation has indicated that the measured fundamental period prior to any damaged/destructive test was around 0.41 s, which is well in the range of the NBCC code formula 0.05*(h_n)^0.75 considering total height of that tested building around 16.98 m [9]. This measured value should be taken with care because the test building was constructed with gypsum wallboard included; while the analyses and modeling performed above were conducted without considering non-structural elements such as gypsum wall board.

Past test result of a six-storey WLFB in UK (TF2000 project) also gave a measured fundamental period of about 0.42 s for the case of the building without non-structural finishes [10]. That study also found that when staircase, plasterboard layer and brickwork layer were included in the measurement, the period was around 0.20s indicating the structure became stiffer.

An attempt was made to check whether the fundamental natural frequency of the multi-storey WLFB could be derived using a single degree of freedom cantilever system with half of the total mass of the structure lumped at the tip. Accounting only for bending effect, the frequency equation for a cantilever beam is [11]:

\[ f = \frac{1}{2\pi} \sqrt{\frac{EI}{mL^4}} \]  

where \( m \) is the total building mass (in this study, \( m = 40,392 \text{kg} \)), \( L \) is the total height of the building (18 m), and \( EI \) is the overall building stiffness estimated from the total deflection (drift) as follows:

\[ EI = \frac{PL^3}{3\Delta_{drift}} \]

By substituting \( P = 107.12 \times 10^3 \text{N} \) (107.12 kN=the base shear), \( H=18 \text{m}, \) and \( \Delta_{drift}=0.07161 \text{m} \), thus \( EI = 2.908 \times 10^9 \text{Nm}^2 \); and using equation (10) \( f = 0.462 \text{Hz} \), or \( T = 2.16 \text{s} \). This value is much higher than that predicted using the stick frame and FE models. Again the discrepancy could be due to the way the modal mass was estimated. This issue needs to be studied further.

8 CONCLUSIONS

Based on an analysis of one six-storey might wood frame building, it has been demonstrated that wood shear wall deflections and their stiffnesses should be calculated for the full height of the building and not on a per storey basis. By incorporating all the effects and configurations of the wood shear walls, an equivalent stick frame model can be assumed. The excellent agreement between the predicted deflection and that obtained from a detailed finite element analysis suggests that the proposed stick frame modelling approach is suitable for use by design engineers for calculating lateral drift of multi-storey light wood frame building. In comparison, the fundamental periods calculated using the stick frame and finite element methods differ substantially, although the latter is similar to that estimated from the NBCC empirical formula. The exploratory work of using a cantilever model to estimate the fundamental period of the building based on the calculated total building drift and mass has also revealed a significant difference between finite element and
cantilever model values. Since the results presented in this paper are limited to one six-storey building, further work is required to substantiate the proposed approach for deflection calculation and to investigate the use of the proposed approach for predicting natural period of multi-storey light wood frame buildings.

With the proposed procedure in the design of wood shear walls, linear dynamic analysis for a wood framed building can be incorporated into the structural analysis, thereby, providing a better understanding of the building’s behaviour under seismic loading condition. Furthermore, with dynamic analysis, torsional effects of a building can be included and need not be avoided by the assumption of flexible diaphragms.

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