Calculation Method for Rigidity and Strength of Roof Diaphragm and Experimental Verification

Tomoaki Soma¹, Masahiro Inayama², Naoto Ando³

ABSTRACT: This report shows calculation method for rigidity and strength of roof with plywood in Japanese post and beam construction. Roof structure deformation mechanism was modelled based on force resistance action of structure members and its joints. Equations for rigidity, yield point and ultimate deformation angle were proposed. In order to verify the equations, shear roof test were examined. Test parameter were kind of roof angle, with or without blocking. Though roof rigidity and strength value per the roof length are set in the housing quality assurance act in Japan, it was confirmed tested specimens have enough values for those. From joint element test, joint of floor joist and beam, load-deformation curves were obtained. These curves were used for proposed calculation for rigidity and strength of roof structure. Calculated result comparing with tested one, both of them had a good coincidence. The calculation method for rigidity and strength of horizontal diaphragm with plywood was valid.

KEYWORDS: Post and beam structure, Horizontal diaphragm, Roof, Wooden roof joist, Wooden beam

1 INTRODUCTION

Japanese wooden roof structure, as shown in Fig.1 has been used in Japan. Though nowadays this structure is main construction method, the roof foundation changed from wooden thin board timber to structural plywood, tie beam changed from log to squared timber.

In this study, calculation method for rigidity and strength of roof with plywood were re-considered, re-arranged and shown. To verify the calculation values, we tested roof shearing strength and rigidity, compared between those values. Experimental parameters were roof angle, nail number (2 or 3) to fasten a rafter on a beam, and with or without rafter cleat.

2 CALCULATION METHOD

2.1 SHEAR RESIST MECHANISM

Fig.2 shows roof structure composed of beams, rafters vertical roof struts with plywood. The consecutive rafter fastened on beam and ridge pole. When in-plane shear force $P$ act to roof structure, angled roof panels above wooden frames keep rectangle shape and resist the shear force. Macro view shear resist mechanism can be shown as followings.

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Whole deformation angle $\theta$ is sum of follow angles

$$\theta = \theta_x + \theta_y + \gamma$$  \(1\)

Where,

$\gamma$: Shear deformation angle of roof plane
$\theta_x$: Angle between pole plate and roof plane
$\theta_y$: Angle between tie beam and roof plane

2.2 RIGIDITY OF ROOF $K_r$

In Fig.2, external force $P$ cause clockwise moment $PH$, the moment balance with resist force by rafter- pole joint in X direction. This force balance is shown as follows.

$$PH = k_x j + k_x \theta$$  \(2\)

Where,

$k_x$: Rigidity of rafter- pole joint in longitudinal direction
$j_x$: Arrangement second moment of rafter- pole joint for neutral axis in X direction

Relationship between longitudinal force of rafter and deformation is shown in Fig.3. The rigidity of horizontal part are shown as eq.(3).

$$\frac{\Delta N}{\Delta x} = \frac{\Delta N \cos \theta}{\delta_x \cos \theta} = k_x$$  \(3\)

Figure 3: Longitudinal force of rafter and deformation

In a similar way, reaction force $PH/L$ in Y direction cause counter clockwise moment $PH$, the moment balance with orthogonal resist force by rafter- pole joint

$$PH = k_x + j_x + \theta_x$$  \(4\)

Where,

$k_x$: Rigidity of rafter- pole joint in orthogonal direction
$j_x$: Arrangement second moment of rafter- pole joint for neutral axis in Y direction

Shear angle of roof plane in horizontal projection $\gamma$ is obtained as follows.

$$PH = k_\gamma \cos \theta \cdot \gamma$$  \(5\)

$k_\gamma$: Rotation rigidity of angled roof plane.

From Fig.4, $k_\gamma$ is shown in eq.(6),(7).

$$k_\gamma = A_\mu \Delta K_\nu - \frac{H - L}{\cos \theta} \Delta K_\delta$$  \(6\)

$$\Delta K_\delta = \frac{1}{k - j_{x}} + \frac{1}{G_y - l}$$  \(7\)

Figure 4: Working force in each part

The relationship between whole deformation angle $R$ and external force moment $PH$ is obtained by eq.(1),(2),(4),(5).

$$R = PH \left( \frac{1}{k_x j_x} + \frac{1}{k_y j_y} + \frac{1}{k_\gamma \cos \theta} \right)$$  \(8\)

Thus, rigidity of roof diaphragm $K_r$ are shown in eq.(9).

$$K_r = \frac{1}{\frac{1}{\Delta K_\delta \cos \theta} + \frac{1}{k_x j_x} + \frac{1}{k_y j_y} + \frac{1}{k_\gamma \cos \theta}}$$  \(9\)

2.3 YIELD AND ULTIMATE STRENGTH

Deformation of roof structure are explained with 3 elements as shown in eq.(1), 1)shear deformation of nail fastening panels to rafter, 2)longitudinal deformation of rafter and 3) orthogonal deformation of rafter. To know yield strength of roof structure, yield strength of each element is needed to obtain.

If external moment $PH$ are divided by arrangement modulus $Z = \frac{I_x}{H^2}$, force working to four corners of panel is obtained. The force equal to longitudinal force of rafter in yield point as eq.(10).

$$PH = Z_x \Delta N_x \cos \theta = \frac{Z_x \Delta N_x \cos \theta}{H}$$  \(10\)

This is divided by $H \cdot L$, shear yield strength per length $P_{\gamma}$, are obtained as shown in eq.(11).

$$P_{\gamma} = \frac{Z_x \Delta N_x \cos \theta}{H}$$  \(11\)

In a similar way, when strength of rafter- pole joint in orthogonal direction reach to yield point $\Delta Q_x$, yield strength of roof structure is obtained as eq.(12).

$$PH = Z_y \Delta Q_y \cos \theta = \frac{Z_y \Delta Q_y \cos \theta}{H}$$  \(12\)

This is divided by $H \cdot L$, shear yield strength per length $P_{\gamma}$, are obtained as shown in eq.(13).

$$P_{\gamma} = \frac{Z_y \Delta Q_y \cos \theta}{H}$$  \(13\)
Yield moment $M_y$ and ultimate moment $M_u$ of angled roof plane are obtained as panel wall theory. External moment working on angled roof plane $M$ is $PH / \cos \theta$. Since this moment reaches to yield point of panel wall $M_y$, following equations are obtained:

$$ \frac{PH}{\cos \theta} = A_w \Delta M_y = \frac{HL}{\cos \theta} \Delta M_y \quad (14) $$

$$ PH = HL \Delta M_y \quad (15) $$

Where,
- $A_w$: Angled roof area
- $\Delta M_y$: Yield moment of angled roof plane

The yield strength per roof length of roof structure $P_y$ is derived as minimum value among $P_{vx}, P_{vj}, \Delta M_y$.

$$ P_y = \min (\Delta M_y, P_{vx}, P_{vj}) \quad (16) $$

As shown in Fig.3, load-deformation curve of rafter-pole joints is assumed by complete elasto-plastic model. Thus, ultimate strength equals to yield’s one. The ultimate strength per roof length of roof structure $P_u$ is derived as minimum value among $P_{vx}, P_{vj}, \Delta M_y$.

$$ P_u = \min (\Delta M_u, P_{vx}, P_{vj}) \quad (17) $$

### 2.4 ULTIMATE DEFORMATION ANGLE AND PLASTICITY MODULUS

(Due to the limit of the available pages, this section is spared.)

### 3 EXPERIMENT

In order to verify the calculation method, we examined shear test of roof structures.

#### 3.1 MATERIAL AND METHOD

Experimental parameters were roof angle, nail number (2 or 3) to fasten a rafter on a beam, and with or without rafter cleat as shown in Table1. Example of specimen is shown in Fig.5, photo of test setup is shown in Fig.6.

**Table 1: Specifications of specimens**

<table>
<thead>
<tr>
<th>angle</th>
<th>nail number of rafter joint</th>
<th>cleat number</th>
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<tbody>
<tr>
<td>tan $\theta$</td>
<td>3/10</td>
<td>3-N75</td>
</tr>
<tr>
<td>a</td>
<td>5/10</td>
<td>2-N75</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>3-N75</td>
</tr>
<tr>
<td>c</td>
<td>10/10</td>
<td>N50</td>
</tr>
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</table>

Horizontal projected size of roof plane was 2730*1820mm. The roof plane was put on wooden flame which was 225mm bigger than the plane in pole direction. Pole – tie beam joint was fasten by M12 bolts. Pitch of rafter was 455mm, structural plywood was nailed by N50 at 150mm pitch. Tie beams, poles and ridge poles were Dagrasfir, rafters were Japanese cider.

#### 3.2 RESULTS AND DISCUSSION

Load- deformation curves obtained in tests are shown in Fig.7 (a) to (e). Rigidity decreased with roof angle increasing. Any specimens were broken at rafter-pole joints or rafter-ridge pole joints in ultimate state. Thus, the factor determines strength was shear resist of nails which fasten rafter to pole of ridge pole, as a result, It would appear that there are no differences among $P_{max}$ of any specimens. Nail number (2 or 3) of rafter joints effected rigidity of specimens. The rigidity in 3 nails joints was higher than one in 2 nails joints. The specimen with 3 nailed rafters joint did not decrease load after its yield deformation. Specimen with rafter cleat was almost same as one without it for rigidity and strength, but that had better energy absorption. As shown in Fig.8, nails pullout by rotation of plywood and rafter sprit by shear force on nails which fasten plywood to rafters in ultimate state were observed.

### 4 DATA ANALYSIS

Characteristic values are shown in Table2. Calculated values are shown in gray lines. To calculate the joints data shown in Table3 was used. The data is obtained in element tests separately.

In roof angle 3/10, 5/10 with 2N-75, 5/10 3N75 (Fig.7 (a), (b), (c)), calculated results corresponded with
Calculated yield strengths were determined by \[\Delta M_i\] in any specimens (Eq. (16)), but significant deformation in rafter–pole joints were observed, actually. It remains possible that the roof specimens yielded at rafter–pole joints in longitudinal direction of rafters. Though in 5/10 with cleat (Fig. 7 (d)) calculated yield and ultimate strength had good correlation with experimental one, calculated rigidity was 30% lower than experimental one. This is because it is assumed that cleat contacted totally to rafters from perpendicular side in the calculation. There was allowance between rafter and cleat actually.

5 CONCLUSIONS

The calculation method for rigidity and strength of roof diaphragm was shown in this report. To verify that, we examined roof shear resist mechanism in actual size. The following results were obtained.

1) Load-deformation curves of angled roof structures were explained by shown method. Calculated values had good correlation with experimental one.

2) Force out of plane which was cursed by dimension effect was observed in the shear tests. We may need to fix the deformation of wooden flame cursed by this force.

### Table 3: Data of rafter-pole Joint

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<th>Perpendicular direction</th>
<th>Longitudinal direction</th>
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<tr>
<td></td>
<td>(k_f) kN/cm</td>
<td>(\delta_f) cm</td>
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<tr>
<td>2-N75</td>
<td>5.87</td>
<td>0.35</td>
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<tr>
<td>3-N75</td>
<td>8.76</td>
<td>0.31</td>
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### Table 2: Characteristic values

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REFERENCES


