Probabilistic Design Method for Timber Joints

Till Vallée\(^1\), Thomas Tannert\(^2\), Frank Lam\(^3\)

**ABSTRACT:** The capacity prediction of timber joints is difficult due to the anisotropic and brittle nature of the material, the complex stress distribution as well as the uncertainties regarding the associated material resistance. This paper describes a probabilistic design method that considers the scale sensitivity of material strength modelled using a Weibull statistical function. The method is applied to predict the capacity of two different types of joints: (i) adhesively bonded joints, which provide an interesting alternative for timber structures and (ii) rounded dovetail wood-to-wood joints, which are a relatively new concept adapted to be processed completely automated. For both types of joint, extensive experimental and numerical investigations were carried out. Tests on small clear specimens were conducted to determine input parameters for finite element analyses. Experimental and numerical results are in good agreement and consequently, the proposed method has immediate application for the improvement of joint design.

**KEYWORDS:** Timber joints, bearing capacity, size effect, strength prediction, probabilistic method

1 INTRODUCTION

The capacity prediction for timber joints is difficult due to the anisotropic and brittle nature of the material, the complex multi-axial stress distribution, and the uncertainties regarding the associated material strength. This paper describes investigations on two different types of joint: (i) adhesively bonded double lap joints and (ii) rounded dovetail wood-to-wood joints. Both types of joints represent efficient methods for connecting structural timber members and are applied in praxis, however, no generally accepted method to predict their capacity is available.

1.1 Adhesively bonded timber joints

Adhesively bonded joints, example shown in Figure 1, exhibit higher strengths and stiffnesses than mechanical connectors, the latter being of particular importance when the governing factor is serviceability limit state. At the same time, bonded joints are more durable since they do not require drilled holes, which facilitate the ingress of moisture. Additional advantages are a favourable behaviour under reversed loading, and a natural protection by sealing off environmental influences. Adhesive bonding can provide an efficient and durable method, both in repair and in new-build applications, provided that the joints are correctly designed using an appropriate structural approach, that suitable materials and specifications are adopted, that the work is done by experienced operatives; and that strict quality control is exercised [1].

Adhesive bonding, although increasingly being used for fibrous and anisotropic materials such as fibre reinforced polymers, e.g. [2], is not yet widely applied for timber structural elements, despite its advantages.

![Figure 1: Adhesively bonded double lap joint](image)

1.2 Rounded dovetail wood-to-wood joints

The Rounded Dovetail Joint (RDJ), named after the rounded shape similar to a dovetail (Figure 2) is adapted to be produced with a CNC-timber processor. A number of experimental studies on RDJ provided valuable insight, revealing that failure under shear loading was typically brittle, and occurred in the elastic range of the load deformation curve, e.g. [3–4]. Although RDJ were shown to be able to undergo large rotations before failing, the brittle nature of joint failure was independent of loading for similar dovetail geometries.
Although there has been significant work on failure criteria development, a general and unified model for timber is not yet available. One issue for the application of any suitable criterion is the inherent large variability of mechanical parameters of timber, especially if considering strength data. This limitation must be addressed before criteria can be reliably used to predict strength of timber joints.

1.4 Size effect in timber

The strength of brittle materials, such as timber, exhibit size effect; this is to say that strength decreases with increasing specimen size under the same test conditions. Three main types of size effects may be distinguished [7]: i) a statistical size effect; ii) an energetic size effect; and iii) the possible size effect due to micro-cracks.

In materials exhibiting statistical size effect, although elastic properties such as the elastic modulus can still be assumed to be volume averaged, failure tends to initiate from flaws, which are randomly distributed within the material volume. For brittle materials like timber, statistically based size effects on strength are adequately explained by probabilistic theories such as the Weibull strength theory [8].

The Weibull distribution is conceptually based on the weakest link theory, which assumes that the material is comprised of small elements linked together and that failure of the material as a whole occurs when one of these elements fails. The cumulative survival probability of a volume $V$ subjected to a non-uniform stress distribution is given by Equation (2):

$$P_s = \exp \left[-\int_{V} \left(\frac{\sigma}{\sigma_0}\right)^k dV \right]$$

where, $\sigma$ is the stress acting over a volume $V$, $\sigma_0$ is the characteristic stress or scale parameter and $k$ is the shape parameter that gives a measure of the strength variability, with low values of $k$ corresponding to a high variability in material properties and large size effects.

One consequence of Equation (2) is that for two volumes $V_1$ and $V_2$ submitted to constant stresses $\sigma_1$ and $\sigma_2$ at failure, assuming equal probabilities of survival, the relationship given in Equation (3) is obtained:

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{k}}$$

Equation (3) allows an implementation of size effects in numerical procedures. The Weibull distribution parameters can be estimated using either the maximum likelihood method or least squares/rank regression etc. Weibull theory has been successfully applied to characterize the magnitude of size effects of timber, if considering brittle failure modes. Further progress has been made towards applications of size effect to a variety of loading conditions and materials, refinements of the statistical basis, and implementation of the theory's predictions to design standards [9].

Figure 2: Rounded dovetail joint

The structural performance can potentially be improved by considering its geometrical features as design parameters to reduce concentration of stresses, if detailed knowledge of the stress distribution within the connection is available.

1.3 Failure modes of timber joints

Failure modes of timber connections depend on member and connection geometry as well as material type and its associated failure modes. In tension and shear, timber essentially exhibits a linear elastic behaviour, and failure is marked by a brittle fracture.

For material failure prediction, criteria based on either continuum mechanics or fracture mechanics exist. The fracture mechanics approach assumes a pre-existent crack and the conditions for crack growth are usually determined by comparing energy release rates with their critical values. The continuum mechanics approach considers the nature and magnitude of stresses or strains, nowadays usually determined using Finite Element Analysis (FEA) and formerly by analytical formulae.

Various failure criteria for wood have been developed, and various in depth reviews were published, e.g. [5]. A commonly applied criterion was proposed by Norris [6], see Equation (1) for the 2D stress state:

$$\begin{align*}
\left(\frac{\sigma_x}{f_x}\right)^2 - \left(\frac{\sigma_x \sigma_y}{f_x f_y}\right) + \left(\frac{\tau_{xy}}{f_{xy}}\right)^2 &= 1 \\
\left(\frac{\sigma_y}{f_y}\right)^2 - \left(\frac{\sigma_x \sigma_y}{f_x f_y}\right) &= 1
\end{align*}$$

(1)

Where $\sigma_x$, $\sigma_y$, and $\tau_{xy}$, are the normal and shear stresses, respectively and $f_x$, $f_y$, $f_{xy}$ are the material strength parameters.
2 EXPERIMENTAL INVESTIGATIONS

2.1 Adhesively bonded timber joints

2.1.1 Specimen description
Symmetrical double-lap joints with rectangular sections were fabricated at the timber and composites laboratory of the Bern University of Applied Sciences, see Figure 1. The joints consisted of two outer and two inner timber boards bonded by a layer of SikaDur330, a stiff and brittle two component epoxy adhesive. To keep the cumulative cross-section constant, the inner adherends were always twice as thick as the outer ones with 38 mm and 19 mm, respectively. The width of the timber and the adhesive layer were kept constant with 50 mm and 1.0 mm, respectively. The overlap length $L$ was varied (40 mm to 280 mm in steps of 40 mm).

2.1.2 Material and methods
The timber species used was Spruce (Picea abies) cut from high quality defect-free boards, conditioned to 12% moisture content prior to specimen manufacturing, and then again stored in constant climate until testing. The mechanical properties required for the numerical investigations were determined on small clear specimens from the same boards used to produce the joints. All experiments were performed in a universal testing machine as quasi-static axial tensile tests under a displacement-controlled rate of 5 mm/s, up to failure load. To avoid local compression due to the low compression strength and stiffness perpendicular to the grain, the specimens had to be cut in dog-bones shapes, to allow for the tensile forces to be introduced. For all specimens, the maximum load ($F_{\text{EXP}}$) was recorded.

2.1.3 Experimental results
All investigated adhesively bonded joints collapsed in a brittle manner. The joints almost always failed by splitting just below the end of the overlap; few specimens failed at the end of the overlaps; but in all cases failure was triggered by a crack that developed from the surface, as illustrated by Figure 3. Failure always occurred in the adherends – in no case did the adhesive layer fail. Joint strength is related to the overlap length: both are positively correlated, shown in Figure 5; however beyond 160 mm overlap length, no significant joint strength increase is noticed.

2.2 Rounded dovetail wood-to-wood joints

2.2.1 Specimen description
A total of ten experimental series consisting of a main beam and a joist connected by RDJ were tested at the Timber Engineering and Applied Mechanics Lab of the University of British Columbia. A control geometry was chosen as: dovetail flange angle 15° and dovetail height 119 mm. Three levels of dovetail heights (109, 129 and 139 mm) and three levels of flange angle (20, 10 and 5°) were investigated. The remaining parameters were kept constant (dovetail width 50 mm, dovetail depth 28 mm, and dovetail angle 15°). These parameter combinations resulted in different dovetail areas; subsequently used as a measure of joint size.

2.2.2 Material and methods
Kiln-dried Western hemlock (Tsuga heterophylla) was used in this study. The average and standard deviation of moisture content were determined as 12.6% and 1.6%, respectively. The average apparent density was 495 kg/m³ with a standard deviation of 46 kg/m³. The RDJ were applied to connect joists to main beams, shown in Figure 2. Since it is the principal loading of RDJ for their practical application, shear loading was chosen. The detailed experimental set up has been described previously [4]. The load was increased up to failure with a constant rate so that failure occurred after approx. six minutes, in accordance to EN-26891. The applied load from the actuator, the force at the support of the free end of the joist were recorded; the force transmitted by the connection was calculated as the difference between the applied load and the load recorded at the free end of the joist. The capacity at rupture of the joint ($F_{\text{EXP}}$), which corresponds to the definition of the ultimate limit state was determined.

2.2.3 Experimental results
Within the range of geometric parameters, loading and support conditions investigated, failure was always brittle, occurred in the elastic range, and initiated at the bottom of the dovetail of the joist member (Figure 4). After some initial slack, caused by alignment issues, the load increased linearly until first cracks developed. Further increases of load were associated with stable crack development and larger displacements until brittle failure occurred at capacity. Figure 6 illustrates the results of the recorded capacities, $F_{\text{EXP}}$, and indicates that capacity does not increase proportionally with joint size.
3 CAPACITY PREDICTION

3.1 Procedure for strength prediction
The successful prediction of the capacity of timber joints relies on the knowledge of:
- The stress-strain state inside the joint
- The failure criterion of the involved materials
- Determination of the Weibull parameters
- A capacity prediction algorithm

The procedure has been addressed in previous investigations [10–12] and will be only summarised.

3.2 The stress-strain state inside the joint
To assist the interpretation of the experimental results obtained, all joints were modelled using the finite element program ANSYS (v11). The timber material was assumed homogeneous and transverse isotropic. Since brittle failure modes initiating within the elastic range dictate the strength of adhesively bonded timber joints, the material was regarded as linear elastic. All calculations were performed at the mean experimentally gathered strength value.

3.2.1 Adhesively bonded timber joints
In previous studies [10], it was shown that 2D instead of 3D modelling of adhesively bonded joints is accurate enough. Two-dimensional 8-node elements were used. Symmetry conditions were used to reduce the modelling to one quarter. In the loci were experimental failure was observed, i.e. the inner adherend’s area just below the end of the overlaps, a very fine mesh was applied.

3.2.2 Rounded dovetail wood-to-wood joints
In this study, 3D modelling and 20-node elements were applied. Weighting result accuracy against computing time, the model was divided into different mesh zones, depending on the stress gradients. The finest mesh was used along the bottom of the dovetail where the highest stresses were expected. To model the contact between joint and main beam, target and contact surface-to-surface elements were used. Initial gaps, caused by geometry differences between the two members, were considered to model the initial alignment behaviour.

3.3 Material strength determination

3.3.1 Adhesively bonded timber joints
To determine the strength parameters needed \( f_X, f_Y, f_{XY} \) to formulate the failure criterion, axial tension tests were performed on dog-bone shaped specimens exhibiting different orientations, \( \alpha \), relative to the grain. Four series were performed: (i) \( 0^\circ \), involving solely \( f_X \) and delivering the axial strength parallel to the grain, \( f_X \); (ii) \( 10^\circ \); (iii) \( 45^\circ \); and (iv) \( 90^\circ \), which only involves \( f_Y \), delivering the strength perpendicular to the grain, \( f_Y \).

A stress transformation procedure described in [11] delivered the following strength parameters: \( f_X = 98.21 \text{ MPa}, \ f_Y = 4.46 \text{ MPa} \) and \( f_{XY} = 13.66 \text{ MPa} \), respectively.

3.3.2 Rounded dovetail wood-to-wood joints
The brittle material strength properties required were determined according to ASTM-D143. These test series were performed on samples exhibiting different stressed volumes. The experimentally determined mean values were \( f_X = 77.4 \text{ MPa}, \ f_Y = 2.92 \text{ MPa} \) and \( f_{XY} = 7.80 \text{ MPa} \), respectively.

3.4 Determination of Weibull parameters
As timber joints fail under a combination of the stresses \( \sigma_1, \sigma_2, \), it is necessary to extend the concept of the Weibull distribution towards stresses acting conjunctly. For this purpose, it was rationally decided to consider the Norris criterion (Equation 1), the stress operator \( \sigma \), can be replaced by a failure criterion, \( \sigma_F \), defined as follows:

\[
\sigma_F^2 = \left( \frac{\sigma_X}{f_X} \right)^2 + \left( \frac{\sigma_Y}{f_Y} \right)^2 + \left( \frac{\tau_{XY}}{f_{XY}} \right)^2
\]  

(4)

In the latter definition, \( \sigma_F \) = 1 defines failure, for which the subscript \( F \) stands, at the investigated scale. To gather the corresponding statistical parameters, all small specimens tested for the brittle material properties were used. For the material used in the study of adhesively bonded joints the parameters were found to equal \( k = 3.717 \), while \( \sigma_{F0} = 1.124 \).

As the specimens used in the study of rounded dovetail joints exhibited different sizes for the different tests, the corresponding failure stresses could not be used in one single series. To overcome this formal issue, all strengths values were first related to an arbitrary reference volume. This procedure resulted in a homogeneous set of data, with \( k = 4.55 \) and \( \sigma_{F0} = 1.122 \), respectively.

3.5 Algorithm for capacity prediction
If the whole joint is idealized as being constituted by \( n \) elements, its survival depends on simultaneous non-failure of all elements. Consequently, if each constituent element \( i \), with a volume \( V_i \), is subjected to \( \sigma_{Fi} \), the probability of survival of the joint is given by:

\[
P_s = \prod_{i=1}^{n} \exp\left[ -\frac{V_i}{V_0} \left( \frac{\phi_{Fi}}{m} \right)^k \right] = \exp\left[ \sum_{i=1}^{n} \frac{V_i}{V_0} \left( \frac{\phi_{Fi}}{m} \right)^k \right]
\]  

(5)

Where \( V_i \) are the volumes of the finite elements, \( V_0 \) is the volume of the joint specimens, \( \phi_{Fi} \) is given by Equation (4). In this research, the stresses needed to formulate \( \phi_{Fi} \) were gathered using numerical analyses. Thus, after having determined all stresses for all elements, all \( \sigma_{Fi} \) are computed, and eventually the corresponding probability of failures, \( P_s \), associated to each element. The global failure is thus defined as the load level, \( \sigma_{pred} \), for which Equation (5) delivers a probability of survival, \( P_s = 0.5 \).

3.6 Results of capacity prediction
This process has been performed for the investigated timber joints; the results for the adhesively bonded joints are displayed in Figure 5 in function of the overlap length. Figure 6 illustrates the results for the rounded dovetail joints in function of the dovetail area.
4 DISCUSSION

The experimental investigations showed that load deformation response of both investigated joint types, the adhesively bonded timber joints and the rounded dovetail joints, is linear until the brittle failure; which makes them a showcase for the application of probabilistic strength prediction methods. The determination of material strength on off-axis tests proved to be rationale and efficient, since it allowed, for a high number of individual specimens, to not only determine the material strength, to assess the Norris failure criterion, but also the accurate determination of the necessary Weibull-parameters. The subsequent application of the probabilistic strength prediction method proved to be sufficiently accurate, as it has been demonstrated for both the adhesively bonded joints and the rounded dovetail joints.

5 CONCLUSIONS

The capacity determination of systems, including joints, exhibiting brittle failure has, for a long time, been considered difficult, and often solved using empirical methods. This paper offers a new approach by implementing probabilistic concepts in an engineering context, which overcome the difficulties raised by the timber’s inherent brittleness and strength variability. The implementation proved to accurately predict joints capacities of two typical brittle joints over a large set of parameters.

REFERENCES


