CROSS LAMINATED TIMBER: A MULTI-LAYER, SHEAR COMPLIANT PLATE AND ITS MECHANICAL BEHAVIOR

Reinhard Stürzenbecher¹, Karin Hofstetter¹, Josef Eberhardsteiner¹

ABSTRACT: Cross Laminated Timber (CLT) is widely used for load bearing applications in timber engineering. Exhibiting a crosswise lay-up of layers of wooden boards, this engineered wood product constitutes a laminated composite with its distinctive mechanical behavior. The orthotropic material behavior of the individual layers and the extraordinarily high shear compliance of the resulting plate put high demands on an appropriate mechanical description of this wood product. Here, an advanced laminated plate theory is applied therefor, delivering accurate results for deformation as well as for stress components also for thick CLT plates under distributed and concentrated loadings, requiring only slightly higher computational costs than conventional plate theories, which are also included here. The actual, laminate-specific deformation behavior and stress state of CLT plates is presented due to an exact solution, delivering the peculiarities of the characteristic mechanical behavior and consequentially proposing advanced plate theories for their structural design.

KEYWORDS: Laminated wood product, Deformation and stress state, Classical and advanced plate theories

1 INTRODUCTION

Cross Laminated Timber (CLT) constitutes an engineered wood product dedicated to structural applications. Made up of ordinary boards, glued together in a cross-layered fashion, the resulting wood product is employed as structural plate suitable for loading in-plane and out-of-plane.

The orthotropic elastic material behavior of wood and the crosswise lay-up result in a quite distinctive deformation behavior of CLT. In mechanical terms, it represents a two-dimensional, multilayer, exceedingly shear compliant, laminated composite with thick and highly anisotropic layers. The high ratio of modulus of elasticity in fiber direction of lengthwise layers and the corresponding transverse shear modulus of the cross-layers (rolling shear modulus) provokes high shear deformations and a zig-zag shaped deformation pattern across the thickness of the plate. Hence, the courses of the transverse shear strains show considerable discontinuities at layer interfaces, while transverse shear stress distributions follow continuous, though strongly nonlinear courses across the plate thickness. In terms of structural design the continuous shear stress distribution is checked against shear strength values, which are considerably different in adjacent layers, distinguishing the relevance of cross layers here. These CLT specific characteristics render an appropriate and accurate description of the mechanical behavior challenging, but for an intensive and safe use of CLT in heavily loaded structural elements this is nevertheless essential.

The focus of this paper is to seek for a plate theory, which is able to reflect the laminate specific mechanical behavior of CLT and delivers accurate deformation and stress components for the structural design at reasonable computational costs. After a brief summary of the theoretical background of the considered plate theories, we first examine the deformation and stress state of surface loaded CLT plates based on an exact solution for the plate bending problem of laminated plates. This enables to depict the specific courses of displacements, strains, and stresses across the plate thickness, and serve as reference for later comparisons of the performance of various plate theories. In particular, we present an advanced plate theory based on the work of Ren [1], which combines accuracy of results with computational efficiency and shows only slightly higher computational costs than Reissner [2,3] or Mindlin [4] plate theory. Comparing the results obtained by Classical Plate Theory (CPT), First Order Shear Deformation Theory (FSDT) and Ren Plate Theory (RPT) with the exact solution delivers their suitable application ranges for structural design. Finally, the capability to capture the complex stress and deformation state of point loaded and of surface loaded and point supported CLT plates is examined.

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1.1 CURRENT DESIGN APPROACHES

Current design models for CLT mainly emerged from the long tradition of using only one-dimensional elements in wood constructions. Thus, they are based on different beam theoretical considerations to a great extent. Application of laminated beam theory is the most straightforward approach [5], which, however, neglects any shear deformations and two-dimensional stress transfer. The \( \gamma \)-method was introduced by Möhler [6] allowing the analysis of compliantly joined beams by introducing a correction factor to the bending stiffness. In this approach, which is strictly only valid for simply supported beams under sinusoidal loading, the cross-lying layers in CLT are considered as shear compliant connections instead of individual layers [7]. The shear analogy method, introduced by Kreuzinger [8,9], is based on the separation of the two parts of overall bending stiffness of CLT: the individual stiffnesses of the lamellae and the stiffness increments by their joint action. These shares of bending stiffness and the shear stiffness of the individual layers of CLT are assigned to two beams showing coupled displacements. Also compliant joints can be easily considered in this method. Schickhofer et al. [10] recommend Timoshenko beam theory for the structural design of CLT. All beam theoretical approaches are not able to account for two-dimensional load transfer and, thus, cannot reflect the load carrying mechanism of a plate. For slender plates with a single pronounced load bearing direction, these effects are rather negligible, and beam theories lead to reasonable results. For moderately thick CLT plates or also for slender plates with point loadings or point supports, though, the current design approaches are not prepared to represent the laminate-specific deformation and stress state accurately. In order to capture plate-specific load carrying mechanisms, at least in an approximate manner, girder grids are built up [11-13], resulting in quite high modeling effort. Two-dimensional approaches are conventionally limited to applications of Kirchhoff [14] or Mindlin [4] plate theories to the laminate. Czaderski et al. [15] performed experiments on thin and slender three-layered CLT plates under different concentrated loadings and compared the results with corresponding quantities obtained from analytical beam and plate considerations as well as from spatial Finite Elements calculations. Considering plate theories, they showed that Mindlin plate theory delivers suitable results for the deflection, but fails to yield accurate normal stresses. This underlines the need for an improved description of the mechanical behavior of CLT plates by more advanced plate theories.

1.2 ADVANCED PLATE THEORIES

Laminated plate theories are originally designed to capture the laminate-specific mechanical characteristics and, thus, to deliver accurate results also for thick plates even under concentrated loadings. Carrera [16] outlines the historical development of laminated plate theories, and further extensive reviews [17-20] show the multiplicity of available theories as well as their accuracy and computational efficiency. A representative thereof is Ren [1] plate theory exhibiting excellent accuracy at only little higher computational costs in comparison to well-known Mindlin plate theory. Thus, it is proposed as a promising tool for the structural design of CLT.

2 METHODS

In the following the main assumptions contained in the various plate theories are reviewed, which establish a transition from the originally three-dimensional elasticity problem to a two-dimensional one. They concern mostly kinematic constraints for the displacement courses across the plate thickness (see Figure 1). The suitability of these constraints is assessed by comparing results obtained with these theories with the exact solution and experimental results, respectively.

2.1 CONVENTIONAL PLATE THEORIES

Prescribing linearity of displacements as well as the normal hypothesis results in the so-called Classical Plate Theory (CPT), commonly known as Kirchhoff Plate Theory [14] for homogeneous plates and Lamination Theory for laminated plates. The resulting displacement field reads as

\[
\begin{align*}
    u(x, y, z) &= u_0 - z \frac{\partial w_0}{\partial x}, \\
    v(x, y, z) &= v_0 - z \frac{\partial w_0}{\partial y}, \\
    w(x, y, z) &= w_0,
\end{align*}
\]

where \( u_0 \), \( v_0 \), and \( w_0 \) denote the displacements in the plate midplane (\( z = 0 \), cf. Figure 1), and are functions of the in-plane coordinates \( x \) and \( y \).

Due to the strict prescription that cross-sections remain normal to the plate midplane also in the deformed configuration, transverse shear deformations are neglected and transverse shear stresses cannot be derived directly from the plate theory. Thus, they can only be calculated quite expensively from the in-plane stresses by integration of the infinitesimal equilibrium equations.

Retaining the assumption of linearity of displacements but discarding the normal hypothesis [2-4] leads to the First Order Shear Deformation Theory (FSDT). Its distinctive displacement field reads as

\[
\begin{align*}
    u(x, y, z) &= u_0 - z \phi_x, \\
    v(x, y, z) &= v_0 - z \phi_y, \\
    w(x, y, z) &= w_0,
\end{align*}
\]

where the cross-sectional rotations, \( \phi_x \) and \( \phi_y \) (cf. Figure 1) appear as own variables independent of the deflection. This enables to consider the shear deflection of the plate in a mean sense. The rotations, which are constant across the plate thickness, lead to constant shear strains. Following Hooke’s law also constant transverse shear stresses are obtained, which clearly contradict boundary
and equilibrium equations. In order to compensate for this in an energetic balance, a shear correction factor was introduced, amounting to the well-known 5/6 for homogeneous plates. With this factor mean transverse shear stresses can be directly calculated from FSDT for homogeneous plates.

Applying FSDT to laminated plates requires calculating appropriate shear correction factors following the formulae of Whitney [21], which strongly depend on the plate lay-up and the material properties of its laminae. They may deviate considerably from the commonly known value. The accuracy of FSDT strongly depends on these factors [22], so that use of 5/6 instead results in distinct errors, as shown in [23].

2.2 LAMINATED PLATE THEORY

Ren Plate Theory (RPT) is a representative of laminated plate theories derived from an appropriate ansatz for the transverse shear stress distribution. Applying Hooke’s law and integration of linear strain-displacement relations finally results in the distinctive deformation field of RPT [1] in layer $k$:

$$u^k(x, y, z) = u_0 - z \cdot \frac{\partial w_0}{\partial x} + A^k(z) \cdot \xi_x + C^k(z) \cdot \eta_x,$$

$$v^k(x, y, z) = v_0 - z \cdot \frac{\partial w_0}{\partial y} + B^k(z) \cdot \xi_y + G^k(z) \cdot \eta_y,$$

$$w^k(x, y, z) = w_0.$$  

The four additional unknown variables, compared to CPT, $\xi_x$, $\xi_y$, $\eta_x$, and $\eta_y$ control the shear deformations. By their multiplication with layerwise functions $A^k(z)$, $B^k(z)$, $C^k(z)$, and $G^k(z)$ describing in a layerwise fashion the course of displacements across the plate thickness, individual deformation characteristics of each layer can be considered with only seven unknown variables, irrespective of the number of layers. Owing to the adequate transverse shear stress assumptions RPT is able to reflect the laminate specific zig-zag deformation course across the plate thickness and shows the advantage that also the transverse shear stresses can be computed directly from the plate theory, at least at positions, where $\sigma_{zz}$ is not predominant, which is disregarded by RPT. Here, excellent accuracy of deformations and stresses is achieved also for thick laminates.

2.3 EXACT PLATE BENDING SOLUTION (exact)

Pagano [24] used an exponential ansatz for the deformation course in the thickness direction for each individual layer of the laminated plate. The corresponding deformation field for the $k$-th layer reads as:

$$u^k(x, y, z) = e^{\xi z} \cdot u_0^k,$$

$$v^k(x, y, z) = e^{\xi z} \cdot v_0^k,$$

$$w^k(x, y, z) = e^{\xi z} \cdot w_0^k.$$  

Combined with the continuity condition of displacements and transverse stresses at layer interfaces, this ansatz enables the exact, analytical solution of the bending problem, meaning that the solution satisfies Hooke’s law, the linear strain-displacement relations and the infinitesimal equilibrium equations in every point of the plate.

Due to the layerwise character, this plate theory gets computationally quite expensive with increasing number of layers. However, it is most suited to act as reference solution and clearly shows that theories assuming polynomial courses of displacements across the thickness will remain approximations.

Figure 1: CLT-5ply plate in undeformed and deformed configuration

3 RESULTS AND DISCUSSION

In this section, the mentioned plate theories and the exact solution are applied to CLT plates with varying geometry and under different loading conditions. First, the laminate-specific deformation and stress courses across the plate thickness are shown for thick CLT plates under surface loading. Then, the described theories are evaluated for point loaded CLT plates, and the results are compared with corresponding experimental values. Finally, the deformation and stress states of surface loaded, point supported slender CLT plates are investigated.

3.1 ORTHOTROPIC MATERIAL BEHAVIOR

In commercial production of CLT mainly spruce boards of quality class C24 according to DIN 1052:2004 are used. Thus, all subsequent calculations are based on the elastic properties of this quality class. The standard only differentiates between material properties parallel and perpendicular to the wood fibers, and so also here no distinction between $R$ and $T$ direction is made as is common practice in CLT. Despite the equal treatment of $R$ and $T$ direction, an orthotropic formulation is adopted, what means in particular that the shear modulus in the transverse plane $G_{RT}$ is not computed from the elastic modulus and the Poisson ratio in this plane, but
constitutes an independent material constant. According to the standard DIN 1052:2004 the modulus parallel to the fiber direction, here denoted as $E_h$, amounts to 11,000 N/mm² for C24, the modulus perpendicular to this direction, $E_v$, to 370 N/mm², and the shear modulus $G_v$ to 690 N/mm². In an orthotropic formulation, $G_v$ represents both shear moduli $G_{LR}$ and $G_{LT}$ of wood and, thus, in terms of plate calculations one in-plane and one out-of-plane shear modulus. For the shear modulus perpendicular to the fiber direction a value of $G_{tt} = 50$ N/mm² is used, which was proposed by Blass and Görlacher [25] and is also included in the technical approvals for CLT products. Since Poisson’s ratios for wood boards are not specified in standards or technical approvals, these were taken from Hearmon [26]. He specifies for spruce wood with an elastic modulus of $E_L = 10,900$ N/mm² the following set of Poisson's ratios: $\nu_{LR} = 0.39$, $\nu_{LT} = 0.49$, and $\nu_{RT} = 0.64$. As no distinction between the orthotropic axes $R$ and $T$ is made, the first two ratios are averaged resulting in $\nu_v = 0.44$ as Poisson ratios in planes parallel to the fibers. Finally, one ends up with the following set of elastic constants for the laminae:

$$
\begin{align*}
E_h &= 11,000 \text{ N/mm}^2 \\
E_v &= 370 \text{ N/mm}^2 \\
G_v &= 690 \text{ N/mm}^2 \\
G_{tt} &= 50 \text{ N/mm}^2 \\
\nu_v &= 0.44 \\
\nu_{tt} &= 0.64
\end{align*}
$$

Considering the ratios of moduli of elasticity and shear moduli, namely $E_h/E_v \approx 30$, $G_v/G_{tt} \approx 14$, and $E_v/G_{tt} \approx 220$, the extraordinarily strong anisotropy of wood is obvious. These ratios underline the necessity of an appropriate material description in the framework of plate theories.

3.2 PLATE GEOMETRY AND BOUNDARY AND LOADING CONDITIONS

The subsequent plate calculations are all performed for three- and five-layer CLT elements denoted as CLT-3ply and CLT-5ply in the following. The thickness of the individual layers is chosen to amount to alternately 26 mm and 40 mm. This leads to more or less balanced laminates in terms of the thickness ratio of lengthwise and crosswise layers for both cases of CLT-3ply [26/40/26] and CLT-5ply [26/40/26/40/26]. This results in total thicknesses $h$ of 92 mm and 158 mm, respectively. The lateral length $a = b$ of the quadratic plates was controlled by the varying slenderness ratio $a/h$.

For the analytical solution of the plate bending problem according to the different plate theories, the Navier type solution was applied. This is applicable to simply supported plates, prescribing the subsequent set of boundary conditions to plates with lateral lengths of $a$ and $b$:

$$
\begin{align*}
at x = 0, a : & \quad v = w = 0 \\
at y = 0, b : & \quad u = w = 0
\end{align*}
$$

The considered surface loads and concentrated loads upon one or more definite subareas are depicted in Figure 2 and were approximated by Fourier series. In order to achieve an appropriate description of the loads, $10 \cdot 10 = 100$ and $25 \cdot 25 = 625$ Fourier terms were considered for both the evenly distributed surface load and for the concentrated load type, respectively.

3.3 CROSS-SECTIONAL WARPING OF CLT

In order to demonstrate the laminate-specific deformation state in terms of the zig-zag shaped courses of in-plane displacements and the discontinuous shear strain distribution across the plate thickness, thick CLT-3ply and CLT-5ply plates exposed to high surface loads of 0.35 N/mm² are examined.

Figure 3 presents the courses of the displacements $u$ and $v$ as well as of the shear strains $\gamma_{xz}$ and $\gamma_{yz}$ at the positions of their maximum values, namely at $(0,b/2,z)$ and $(a/2,0,z)$ respectively. The zig-zag shaped deformation course is most pronounced for the in-plane displacement $u$ of both three- and five-layer CLT. This specific course results from the high shear strains in the cross-layers, which exhibit only low shear modulus $G_{tt}$ in the $xz$-plane. Regarding the displacement $v$ of CLT-3ply, there is almost no zig-zag pattern observed. The middle layer shows high bending stiffness in the $y$-direction, so that the soft surface layers just follow the displacement predominantly forced by the middle layer.
Figure 3: Displacement and shear strain courses across the thickness of CLT-3ply and CLT-5ply plates

The same holds for both surface layers of the CLT-5ply plate, which also do not provoke a distinctive kink in the course of the displacement \( v \). Here, only the middle layer exhibits high shear strains and, thus, gives rise to the typical zig-zag shape of the cross-section. Evaluating the performance of different plate theories and comparing their results with the exact solution, it is obvious that the conventional plate theories such as CPT and FSDT are not able to reproduce the zig-zag shaped displacement courses. RPT is capable to deliver accurate prognoses of both displacements \( u \) and \( v \).

The two out-of-plane shear strains \( \gamma_{xz} \) and \( \gamma_{yz} \) clearly illustrate the different kinematic constraints underlying the investigated plate theories (see Figure 3). CPT neglects shear deformation at all, and so the out-of-plane shear components vanish. FSDT accounts for shear deformations in a mean sense, which is still not sufficient to capture the high differences of shear strains in subsequent layers of CLT plates. The results of Ren plate theory match closely the exact solution indicating the proper consideration of out-of-plane shear deformations. In view of the discrepancies of out-of-plane shear strains obtained with FSDT, using Hooke's law to calculate the corresponding shear stresses will yield unrealistic results. Thus, also for FSDT the integration of in-plane stresses has to be applied in order to calculate the corresponding shear stresses \( \tau_{xz} \) and \( \tau_{yz} \).

3.4 STRESS DISTRIBUTIONS ACROSS THE PLATE THICKNESS

The stress distribution across the plate thickness is also investigated for thick \( (a/h=10) \) CLT-3ply and CLT-5ply plates under surface loads of 0.35 N/mm². The considered normal stress components \( \sigma_{xx} \) and \( \sigma_{yy} \) and shear stress components \( \tau_{xz} \) and \( \tau_{yz} \) are plotted along the thickness coordinate \( z \) at in-plane positions \((a/2,h/2),(0,h/2),\) and \((a/2,0)\) respectively, where these components reach their maximum values.

For all theories the two in-plane stress components \( \sigma_{xx} \) and \( \sigma_{yy} \) exhibit the correct characteristic shape with discontinuities at the layer interfaces and considerably different slopes in soft and stiff layers. Nevertheless, there are considerable deviations from the exact values. With increasing slenderness of the plates, these deviations decrease, and also results calculated with CPT and FSDT converge to the exact solution then, as shown in the subsequent subsection.

The transverse shear stress distributions obtained from the exact solution exhibit their laminate-specific courses. In order to comply with the stress equilibrium conditions, they are continuous at layer interfaces. The slope of the course is an indicator of the corresponding shear stiffness of the layers. The slight asymmetry of transverse shear stress distributions obtained in the exact solution with respect to the plate midplane origins from the effect of the transverse normal stress \( \sigma_{zz} \), which is disregarded in all other plate theories. As shown in Figure 4, the maximum of transverse shear stresses does not necessarily occur in the plate midplane in thick laminated plates. Nevertheless, increasing the slenderness of the plate, the position of the maximum transverse shear stresses approaches \( z = 0 \), and therefore these stress components were evaluated there in the convergence study in the following subsection.

As discussed above, determination of transverse shear stresses in the framework of CPT and FSDT requires integration of the infinitesimal equilibrium equations, starting from the previously computed in-plane stresses.

![Graphs showing displacement and shear strain courses across the thickness of CLT plates](image-url)
Figure 4: Stress distributions across the thickness of CLT-3ply and CLT-5ply plates

Here, RPT delivers reasonably accurate transverse shear stresses in a direct way without the need of further calculations. In thick plates, the actual transverse shear stress distribution shows local bumps in lengthwise layers, which are neither well reflected by RPT nor by CPT or FSDT, the latter two with an integration of the equilibrium equations.

3.5 CONVERGENCE OF PLATE THEORIES TO THE EXACT SOLUTION

The zig-zag shaped deformation behavior shown in Subsection 3.3 is mainly caused by shear deformations, which are most pronounced in thick plates under evenly distributed loading. Increasing the plate slenderness \( a/h \), the bending deformations become dominant, and finally the contributions of shear deformations to the in-plane displacements as well as their zig-zag shaped course across the plate thickness are hardly recognizable. Thus, with increasing \( a/h \)-ratio, all considered plate theories yield results converging to the exact solution. The convergence rate strongly depends on the elastic properties of the individual layers, in particular on the ratio of modulus of elasticity and shear modulus in the transverse plane. Since for isotropic, homogeneous plates the ratio \( E/G \) is limited to three, CPT and FSDT results converge rapidly to exact values there. For CLT plates, though, the ratio of modulus of elasticity of the lengthwise layers to the corresponding shear modulus of the cross layers, \( E_l/G_{tt} \), amounts to 11,000/50 = 220, causing a by far slower convergence rate of plate theories to the exact solution.

The convergence behavior of the plate theories towards the exact solution was studied on the basis of CLT-5ply plates. The same loadings were applied as in the previous studies in order to enable comparability. The results of the deflection \( w \), the in-plane stress components \( \sigma_{xx}, \sigma_{yy} \), and \( \tau_{xy} \), and the transverse shear stress components \( \tau_{xz} \) and \( \tau_{yz} \) are evaluated at characteristic points (specified in corresponding labels), where each specific component reaches its maximum value. The results are presented in relation to the exact solution (serving as reference value = 100 %) and are plotted over the \( a/h \)-ratio in Figure 5.

Considering the deflection \( w \), CPT results in gross underestimation and exhibits a very slow convergence rate to the exact solution at increasing plate slenderness. Both plate theories accounting for shear deformations, FSDT and RPT, yield reasonably accurate results also for thick plates. For all three in-plane stress components, RPT delivers excellent accuracy. Its results closely approach corresponding exact values already at moderate plate slenderness. Here, the convergence rate of both common plate theories is rather moderate and, unexpectedly, FSDT provides poorer approximations of \( \sigma_{xx} \) than CPT, also with an appropriately calculated shear correction factor. For CPT and FSDT the transverse shear stresses were calculated by integration of the infinitesimal equilibrium equations and show a fast convergence rate for \( \tau_{xz} \) and a moderate one for \( \tau_{yz} \) in increasingly slender plates. As shown in Figure 4, the distribution of the stress component \( \tau_{xz} \) across the thickness shows, primarily at thick plates, a distinctive local bump in the middle layer. This is captured by none of the considered plate theories as mentioned above and results in a moderate convergence rate of also RPT results here. Considering the transverse shear stress component \( \tau_{yz} \), though, RPT delivers high accuracy also for very thick plates.
3.6 CLT UNDER CONCENTRATED LOADING

In order to assess the suitability of plate theory calculations also for plates under concentrated loadings, experimental results are evaluated for comparison. Czaderski et al. [15] performed plate bending experiments on thin and slender three-layer CLT plates with an $a/h$-ratio of 35 and a total thickness of 70 mm. They investigated the mechanical behavior of two lay-ups and of three types of concentrated loadings, namely concentrated loadings on subareas of $150 \times 150\,\text{mm}^2$ centered at the plate midpoint ($mp$), at the plate quarter point ($qp$), and at all four quarter points ($4qp$; cf. Table 1 and Figure 2). For a mean estimate of the elastic properties they performed ultrasonic measurements on the individual layers prior to plate fabrication. Eight different sets of plate layup, loading, and raw material properties were investigated (cf. Table 1), and three replicates of each test set were tested. Depending on the test configuration the deflection was measured at or close to the plate midpoint or quarter point. Moreover, the normal strains on the plate lower surface directly under the load were determined. From these normal strains the normal stress component $\sigma_{xx}$ was calculated using mean elastic moduli of the bottom layers obtained in the ultrasonic measurements. For this purpose the additionally required elastic constants were derived from stiffness ratios from literature.

As for the calculations by the plate theories, 25 Fourier terms were used for the representation of the concentrated loadings in the framework of Fourier expansion for each in-plane direction. Further, all plate calculations were evaluated at the respective measurement points in the experiments. Therefore, the results obtained by Mindlin plate theory (FSDT) here are slightly different to the values published by Czaderski et al. [15], who performed also calculations but with only 13 or 15 Fourier terms in each direction. The experimental data for the deflection $w$ and the stress component $\sigma_{xx}$ are summarized in Table 1 in terms of the mean values of the measurements on the three replicates of each plate set. Further the results obtained with FSDT, RPT and the exact solution are specified in absolute and relative values, where the experimental data serve as reference.

In terms of the deflection $w$, both plate theories as well as the exact solution show a very good agreement with the experimental values. Here, the plate set No. 7 shows the highest deviation from all considered theories amounting to about 7.5%. Considering the experimental results for the normal stress component $\sigma_{xx}$, though, FSDT results in an underestimation of about 20%, even for very slender plates with an $a/h$-ratio of 35. This indicates that the convergence to the exact solution shown in Figure 5 for evenly distributed load is not that fast for concentrated loadings. Hence for stress considerations, application of more advanced plate theories is necessary. RPT and the exact solution are in general able to reflect also the stress state directly under the concentrated loadings. As for $\sigma_{xx}$, considerable deviations of the results obtained with RPT and the exact solution from the experimental data are again only observed for plate set No. 7. The plates of this set showed some variation of the elastic moduli measured at the bottom layers. The local value used for the stress computation was possibly much lower than the mean one, which was taken for the plate calculations. This difference might cause this observed deviations.

On the whole, numerical results obtained with RPT and the exact solution show a good agreement with corresponding experimental results for deflections $w$ as
CLT plates with concentrated loads are generally suitable for normal stresses. Such plates are considered next.

Regarding structural design, dimensions of wide-span supported CLT plates, however, generally maximum stresses are decisive. Such plates are considered next.

3.7 STRESS AND DEFORMATIONS OF SURFACE LOADED, POINT-SUPPORTED CLT PLATES

A wide-span, slender CLT-5ply plate is examined with an $a/h$-ratio of 40. At the edges the plate is simply supported, and in the plate midpoint a point support on an area of $250 \times 250 \text{ mm}^2$ is considered. Since the Navier-type analytical solution prevents geometrical boundary conditions within the plate, this point support was realized by a distributed loading pointing upwards, which was adjusted such that the deflection $w$ is zero there. In the practical calculations, this load at the support served as starting point. It was set to the proposed maximum normal stress perpendicular to the fiber direction for CLT-plates amounting to 3 N/mm². Thereon, the surface load was adjusted in order to realize the constraint of zero deflection in the plate midpoint. This results in an evenly distributed loading of 14.095 kN/m² on the top surface. As the stress distribution at and close to the point support is of prime interest, also here the analytical approach was applied, which, in contrast to finite element calculations does not yield stress singularities at the edges of the point support. In order to reasonably represent a concentrated load on a small subarea of a wide-span plate, in the framework of a Fourier expansion, 100 Fourier terms in each in-plane direction are necessary, indicating the remarkable computational costs here. For FSDT and RPT the surface load and the constraint of zero deflection at the point support were kept, while the resulting reaction force at the point support was adjusted. Thus, normal stresses of 3.233 N/mm², 2.963 N/mm², and 3.005 N/mm² are obtained at the support area for CPT, FSDT, and RPT respectively.

Figure 6 shows the deflection $w$ as well as normal and shear stress components along the $x$- and the $z$- axes. The deflection $w$, evaluated at the bottom surface ($z = h/2$) at $y = h/2$, exhibits the expected course along the $x$-axis with zero deflection at the plate edges and at the support point in the plate middle. Here, results obtained with FSDT and RPT are in good agreement with the exact solution. CPT results, though, underestimate the deflection by a factor of two, clearly pointing out the relevance of shear deformations here.

Figures 6b-d make evident that both commonly known plate theories (CPT and FSDT) lead to underestimation of these stress components. Unexpectedly, CPT delivers more accurate values for $\sigma_{xz}$ than FSDT, while the results of CPT and FSDT are almost the same for $\sigma_{xy}$. Normal stress components obtained by RPT are in reasonably good agreement with the exact solution, delivering some overestimation at the plate top surface and some underestimation directly at the support on the bottom surface. Considering the transverse shear stress components $\tau_{xz}$ and $\tau_{xy}$, their maxima occur at the edge of the subarea where the reaction force is effective. The corresponding in-plane positions are $(a/2, h/2-125)$ and

<table>
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<th>No.</th>
<th>lay-up</th>
<th>$E_l$</th>
<th>load</th>
<th>$w$ [mm]</th>
<th>deviation [%]</th>
<th>$\sigma_{xx}$ [N/mm²]</th>
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<td>mp</td>
<td>18.2</td>
<td>5.26</td>
<td>19.16</td>
<td>19.15</td>
</tr>
<tr>
<td>7</td>
<td>10/50/10</td>
<td>10,000</td>
<td>qp</td>
<td>17.2</td>
<td>7.36</td>
<td>18.47</td>
<td>18.50</td>
</tr>
<tr>
<td>8</td>
<td>10/50/10</td>
<td>12,000</td>
<td>qp</td>
<td>15.0</td>
<td>2.28</td>
<td>15.34</td>
<td>15.36</td>
</tr>
</tbody>
</table>

Table 1: Experimental values from Czaderski et al. [15] and results from different plate theories
Figure 6: Deformation and stress state of a surface loaded, point supported CLT-5-ply plate

(a/2-125,b/2) for \(\tau_{xz}\) and \(\tau_{yz}\), respectively. The exact solution indicates the actual shear stress distributions across the plate thickness at these positions, which are now, due to the pronounced effect of the normal stress component \(\sigma_{zz}\), quite different from the shear stress distributions depicted in Figure 4. Here, the transverse shear stress distribution shows pronounced bumps in layers with high shear stiffness in the plane of the examined shear stress. The maximum thereof emerge in the respective layer closest to the support. As all considered plate theories disregard the normal stress component \(\sigma_{zz}\), they are not able to reproduce asymmetric transverse shear stress distributions. Also integrating equilibrium equations based on in-plane stresses obtained with CPT or FSDT delivers results, which do not represent the transverse shear stress distribution across the plate thickness correctly. The assumed shear stress distributions in RPT are not able to reproduce the particular shapes of \(\tau_{xz}\) and \(\tau_{yz}\) either. Integration of in-plane stresses obtained by RPT, though, leads to accurate prognoses of transverse shear stresses both in qualitative and quantitative respect. As for \(\sigma_{yy}\), CPT and FSDT deliver almost the same values for \(\tau_{yz}\).

4 CONCLUSIONS

Cross Laminated Timber (CLT) is widely used as structural plate loaded both in in-plane and out-of-plane direction. In this paper, CLT is seen as a representative of laminated composites plate with its distinctive characteristics and deformation behavior. The crosswise lay-up, the highly anisotropic material behavior of the layers, and the extraordinarily low shear stiffness render this wood product more demanding in terms of mechanical considerations than others.

The aim of this paper is to present a plate theory, which is able to reflect the laminate-specific mechanical behavior of CLT and delivers accurate deformation and stress components for the structural design of CLT at reasonable computational costs. Based on an exact solution for the plate bending problem of laminated composites, the actual deformation and stress state of CLT plates under several loadings are examined, highlighting the laminate-specific courses of displacements, strains, and stresses across the plate thickness. Comparison of plate theory calculations with the exact solution clearly exhibits the effect of the kinematical constraints involved in the various theories. The commonly known plate theories by Kirchhoff and Mindlin are not able to reproduce the zig-zag shaped deformation courses and, moreover, result in underestimation of in-plane stresses for a wide range of plate geometries. In order to obtain suitable results for the transverse shear stresses from these theories, integration of in-plane stresses is necessary here. The laminated plate theory derived by Ren is based on appropriate assumptions for the distribution of transverse shear stresses across the thickness of a laminated plate and exhibits a displacement field with only two more solution variables than the well-known Mindlin plate theory. Having only slightly higher computational costs, Ren plate theory delivers excellent agreement of computed deformations and stress components with corresponding exact values also for thick plates, rendering it a reliable design tool for CLT for a wide range of applications.

Experimental results of slender point loaded CLT plates clearly show the necessity of employing advanced plate theory calculations for suitable stress determination. Here commonly known plate theories are not able to
reflect the stress state at and in the vicinity of the loading. Even more challenging is the mechanical behavior of a wide-span CLT plate with a point support in the plate middle. Here an advanced laminated plate theory is indispensable for reliable deformation and stress prognosis. This points out that not only the plate slenderness, but also geometrical boundary conditions as well as loading conditions have pronounced influences on the accuracy of plate calculation results.

Considering the laminate-specific mechanical characteristics of CLT, Ren plate theory is proposed for structural design delivering suitable deformation and stress prognoses. Further, the knowledge of the actual deformation and stress state as well as reliable plate theories are intended to support the favorable development of this high-performance wood product and to strengthen its competitiveness against other mass building materials.

REFERENCES