Estimation of shear modulus by FEM bending simulation of steel-plate-inserted glulam wood beams

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Summary
Shear stiffness of glulam beams is very small compared with it’s bending stiffness. While hybridization of glulam beams by inserted steel plates improves the bending stiffness, it does not so improve the shear stiffness. As a result, ratio of shear deformation to bending deformation is greater in the steel-plate-inserted glulam beams than even in glulam-only beams. Therefore it is very important to estimate shear modulus (including shear coefficient) for the steel-plate-inserted glulam beams. In this study we numerically, analytically and experimentally estimate the shear modulus (or coefficient) of the steel-plate-inserted glulam beams. We discuss the validity and accuracy of each estimation method.

1. Introduction
The shear modulus of glulam beams is generally about $1/12 ~ 1/16$[1] of its out-of-plane bending Young’s modulus. Recently also in Japan use of glulam beams is increasing, as hybridization technique between glulams and steel plates is developing. While hybridization of glulam beams by inserted steel plates improves the bending stiffness, it does not so improve the shear stiffness. As a result, ratio of shear deformation to bending deformation is greater in the steel-plate-inserted glulam beams than even in glulam-only beams. We can estimate the degree of the shear deformation by Timoshenko’s beam theory, but we need to know the shear modulus (including shear coefficient) of the glulam/steel composite beams. Although shear coefficients for isotropic material beams can be derived analytically from 3-dimensional elastic theory, it is difficult to derive those for composite beams. In this study we try to estimate the shear moduli or coefficients for the steel-plate-inserted glulam beams analytically, numerically and experimentally. In the analytical approach we carry out the integral of the first sectional moment function using reduced properties to glulams for the composite section. In the numerical and experimental approaches we back-calculate the shear moduli or coefficients from the relations between loads and deflections using Timoshenko’s beam equation. We discuss the validity and accuracy of each estimation method.

2. Models
We model the four types of beams (one glulam beam and three types of steel-plate-inserted glulam beams) as shown in Fig. 1 and make them as experimental specimens. Two pieces are made for each type beam. We denote the each type by 0%, 15%, 25%, 35% corresponding to the depth of the steel plates. The steel plates are bonded by epoxy resin. The average axial Yong’s modulus $E_w$ and the average shear modulus $G_w$ of the eight raw glulam beams (not steel-inserted yet) measured by bending tests (ASTM, D198-94)[2] are 7.49GPa and 0.442GPa respectively.
above, we define the shear coefficient of the composite beams as follows.

\[ k_w = \frac{P\ell}{4(G_wA_w)\left\{v - \frac{P\ell^3}{48(E_wI_w + E_sI_s)}\right\}} \]  

(2)

For comparison, we also define the shear coefficient of the composite beams \( k_n \) for the case using \( G_wA_w + G_sA_s \) as shear stiffness.

\[ k_n = \frac{P\ell}{4(G_wA_w + G_sA_s)\left\{v - \frac{P\ell^3}{48(E_wI_w + E_sI_s)}\right\}} \]  

(3)

3. Numerical approach

3.1 Shear Coefficient

The central deflection \( v \) of simply supported beam subjected to central concentrated load \( P \) is given by the following Timoshenko’s beam equation[2, 3, 4]:

\[ v = \frac{P\ell^3}{48EI} + \frac{P\ell}{4kGA} \]  

(1)

where \( \ell \) is the axial length of the beam; \( E \) is the axial Young’s modulus, \( I \) is the moment of inertia; \( k \) is the shear coefficient; \( G \) is the shear modulus; \( A \) is the cross-sectional area; respectively. We denote the properties of the glulam(wood) part by the subscript \( w \) and the properties of the steel part by the subscript \( s \). While the bending stiffness of the composite beam \( EI \) can be given by the sum of the stiffness of each material part as \( E_wI_w + E_sI_s \), the shear stiffness of the composite beam \( GA \) can not be given as \( G_wA_w + G_sA_s \). The relative errors of the central deflections to FEM solutions for 1)the elementary beam solutions \( (v = \frac{P\ell^3}{48EI}) \), 2)Timoshenko’s beam solutions with \( kGA = \frac{2}{3}G_wA \) in the equation (1) and 3)Timoshenko’s beam solutions with \( kGA = \frac{5}{6}(G_wA_w + G_sA_s) \) in the equation (1) are shown in Fig. 2. The errors of Timoshenko’s beam solutions with \( \frac{5}{6}G_wA \) to FEM are within about \( \pm 5\% \) when the depth of the inserted steel plate is less than \( 40\% \), while the errors of Timoshenko’s beam solutions with \( \frac{5}{6}(G_wA_w + G_sA_s) \) are larger especially where the steel plate depth is around \( 25\% \). Since the shear stiffness of this type composite beams can be approximated by \( G_wA \) as mentioned above, we define the shear coefficient of the composite beams \( k_w \) as follows.

\[ k_w = \frac{P\ell}{4(G_wA_w)\left\{v - \frac{P\ell^3}{48(E_wI_w + E_sI_s)}\right\}} \]  

(2)

For comparison, we also define the shear coefficient of the composite beams \( k_n \) for the case using \( G_wA_w + G_sA_s \) as shear stiffness.

\[ k_n = \frac{P\ell}{4(G_wA_w + G_sA_s)\left\{v - \frac{P\ell^3}{48(E_wI_w + E_sI_s)}\right\}} \]  

(3)
3.2 FEM simulation

In this subsection we carry out FEM analyses for the composite beams under bending by 8-node isoparametric brick element and back-calculate the shear coefficients $k_w$ and $k_n$, substituting loads and the obtained deflections by FEM into the equations (2) and (3) [3, 4]. The used FEM tool is GPL free soft CalculiX[5]. Although we adopt 3-point bending of simply supported beam for the back-calculations of $k_w$ and $k_n$ as shown in the equations (2) and (3), here we numerically analyze the cantilever beam model which can be regarded as the half of the simply supported beam in order to save the number of the solid elements. We denote the axes of the width, depth and axial directions of the beam by $x$, $y$ and $z$ respectively as shown in Fig. 3, where $b$, $h$ and $\ell$ denotes the width, depth and axial length of the beam respectively. We analyze the half of the beam divided by the vertical center plane $yz$ under the condition of the symmetry. The half width $b/2$, the depth $h$ and the axial length $\ell$ are divided into the number of elements 6, 30 and 120 respectively. Boundary conditions are given by restriction of nodal displacements. $z$-displacements on the fixed end plane $xy$, $y$-displacements on $x$-axis and $x$-displacements on the symmetry plane $yz$ are restricted. The tip load $P_2$ is distributed uniformly on each node of the free end [6]. The properties of the glulam materials are as follows; axial Young’s modulus $E_z = 7.49$GPa; Young’s moduli perpendicular to axis $E_x = E_y = E_z/25$GPa; shear moduli $G_{xy} = G_{xz} = G_{yz} = 0.442$GPa; Poisson’s ratios $\nu_{xy} = \nu_{xz} = \nu_{yz} = \nu_{yx} = 0.016$; $\nu_{xz} = \nu_{yz} = 0.4$ so that the strain-stress matrix becomes symmetric. The properties of the steel plates are as follows; Young’s modulus $E = 210$GPa; Poisson’s ratio $\nu = 0.3$. The influences of the shear deformation caused by the steel plate insertion is shown in Fig. 4. The vertical axis indicates the error of deflections solved by FEM to the elementary beam solution ($\varepsilon = P \ell^3 / 48EI$) and the horizontal axis indicates the span/depth ratio($\ell/h$). The errors indicated by the vertical axis approximately means how large shear deflections are compared to their bending deflections. The shear deflections of steel-plate-inserted glulam beams are two times or more larger than glulam-only beams. The shear deflections of the beams with 25%-depth plate are larger than 15%-depth and even than 35%-depth, as Fig. 2 shows that the shear deflection (error of elementary beam solution to FEM) is largest around 25%-depth. The shear coefficients $k_w$ derived by substitution of deflections calculated by FEM into the equation (2) are shown in Fig. 5. As the plate depth or the span/depth ratio is getting larger, the shear coefficients are getting larger. In order to confirm the influence of the error between $G_wA$ and $GA$ as composite section, we show in Fig. 6 also the shear modulus $kG$ derived by substitution of deflections calculated by FEM into the equation (1). The appearance of the both curves are almost same and it seems that the error between $G_wA$ and $GA$ is small. For comparison, the shear coefficients $k_n$ calculated by the equation (3) are shown in Fig. 7. As the plate depth is getting larger, the shear coefficients are getting...
smaller, opposite to the case of $k_w$.

4. **Analytical approach**

Shear coefficient of isotropic material beams with open cross section is given by the following equation[7]:

$$k = \frac{t^2 I^2}{A \int_A Q(y)^2 dA} \quad (4)$$

where $t$ is the beam width; $Q(y)$ is the first sectional moment function defined by $Q(y) = \int_{y_e}^y y t dy$, where $y_e$ is the distance between the beam edge and the horizontal axis on the centroid. We try to derive the shear coefficient of the composite beam as shown in Fig. 8 using the equation (4). Denoting the ratio of Young’s modulus of the steel plates to the glulams by $n = \frac{E_s}{E_w}$, we define the reduced moment of inertia $I_n$ and the reduced area $A_n$ of the composite section to the glulam as follows.

$$I_n = \frac{E_w I_w + E_s I_s}{E_w} = I_w + n I_s \quad (5)$$

$$A_n = \frac{E_w A_w + E_s A_s}{E_w} = A_w + n A_s \quad (6)$$
Rewriting \( A_n \) as the following equation, we define the reduced width of the part with the inserted steel plate to the glulam \( b_n \).

\[
A_n = \frac{E_w (b - t_s) h_s + E_s t_s h_s}{E_w} = \{(b - t_s) + nt_s\} h_s = b_n h_s
\]

(7)

The first sectional moment function \( Q_s(y) \) for the upper part with the inserted steel plate \((-\frac{h_s}{2} < y < -y_w)\) is given by the following equation.

\[
Q_s(y) = \int_{-\frac{h_s}{2}}^{-y} yb_n dy = b_n \left(\frac{y^2}{2} - \frac{h_s^2}{4}\right) - \frac{b_n}{2} \left(y^2 - \frac{h_s^2}{4}\right)
\]

(8)

The first sectional moment function \( Q_w(y) \) for the upper part without the inserted steel plate \((-y_w < y < 0)\) is given by the following equation.

\[
Q_w(y) = Q_s(-y_w) + \int_{-y_w}^{y} ybdy = b_n \left(\frac{y^2}{2} - \frac{h_s^2}{4}\right) + \frac{b}{2} \left(y^2 - y_w^2\right)
\]

(9)

The shear coefficient of the composite section is given by the following equation[7].

\[
k = \frac{b^2 I_n^2}{A \int_A Q(y)^2 dA} = \frac{b^2 I_n^2}{2A \left(\int_{-\frac{h_s}{2}}^{-y_w} Q_s^2(y) bny dy + \int_{-y_w}^{0} Q_w(y)^2 bdy\right)}
\]

(10)

Which sectional area among \( A_n \), \( A_w \) and \( A \) is the most suitable for \( \bar{A} \) depends on what one regards as composite section. Here we compare each case. The results of the integrations in the equation (10) are given as follows.

\[
\int_{-\frac{h_s}{2}}^{-y_w} Q_s^2(y) bdy = \frac{b_n^2 b}{4} \left(-\frac{y_w^5}{5} + \frac{h_s^2}{6} y_w^3 - \frac{h_s^4}{16} y_w + \frac{h_s^5}{60}\right)
\]

(11)

\[
\int_{-y_w}^{0} Q_w(y)^2 bdy = \frac{b_n^2 b}{4} \left(y_w^5 - \frac{h_s^2}{2} y_w^3 + \frac{h_s^4}{16} y_w \right) - \frac{b_n^2 b}{3} \left(y_w^3 - \frac{h_s^2}{4}\right) y_w^2 + \frac{2}{15} b^3 y_w^5
\]

(12)

The shear coefficients calculated from the equation (10) for the composite beam models are shown in Table 1. When the plate depth ratio \( \frac{h_s}{b} \) is 0%, the shear coefficient is 0.833(\( \frac{5}{6}\)) for any sectional area. If we set \( E_w = E_s \), the shear coefficient is 0.833(\( \frac{5}{6}\)) for any plate depth. While numerically estimated shear coefficients shown in Fig. 5 are getting larger along with the increase of the plate depth, the calculated shear coefficients from the equation (4) are getting smaller. In that sense the relation between the analytically estimated shear coefficients and the plate depth is rather close to that of the numerically estimated coefficients \( k_n \) shown in Fig. 7.
Table 1  Shear coefficient $k (\frac{1}{k})$

<table>
<thead>
<tr>
<th>$A_n$</th>
<th>$A_w$</th>
<th>$A$</th>
<th>$E_w = E_s$ ($A_n = A$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.833</td>
<td>(1.20)</td>
<td>0.833</td>
<td>(1.20)</td>
</tr>
<tr>
<td>0.462</td>
<td>(2.16)</td>
<td>0.760</td>
<td>(1.31)</td>
</tr>
<tr>
<td>0.281</td>
<td>(3.55)</td>
<td>0.589</td>
<td>(1.70)</td>
</tr>
<tr>
<td>0.174</td>
<td>(5.75)</td>
<td>0.444</td>
<td>(2.25)</td>
</tr>
</tbody>
</table>

Fig. 9  Estimation of $E$ and $kG$ (No.5, 25%)

5. Experimental approach

We estimate shear modulus $kG$ of the composite beams by 3-point bending tests. The typical relation curve between $\frac{1}{E_b}$ and $(\frac{h}{l})^2$ are shown in Fig. 9, where $E_b$ is the bending Young’s modulus derived by substitution of the measured deflections into the elementary beam solution ($v = \frac{P l^3}{48 E_b I}$) and the vertical intercept of the regression line gives the axial Young’s Modulus $E$, while the slope of the regression line gives $\frac{1}{kG}$ [2]. The estimated shear modulus $kG$ and bending stiffness $EI$ from the graphs such as Fig. 9 are shown in Table 2, where $E_w$ in $E_w I_w + E_s I_s$ and $G_w$ are axial Young’s modulus and shear modulus measured for each beam before plate insertion. $EI$ is given as the multiplication between measured $E$ from the intercept in the graph and $I$ for the rectangular section. The estimated values of $EI$ are close to the values just anticipated as $E_w I_w + E_s I_s$ and are getting larger along with the increase of the plate depth. On the other hand, estimated shear modulus $kG$ has correlation neither with the increase of the plate depth nor with varying values of shear moduli of the glulam part $G_w$. Although it seems that the accuracy of the bending test is not so bad, as judged from the correlation between $EI$ and $E_w I_w + E_s I_s$, there might be possibility that shear stiffnesses delicately depend on the bonding conditions between plates and glulams or that slit cutting for plate insertion change the shear elasticity of glulam beams.

Table 2  Experimentally estimated properties

<table>
<thead>
<tr>
<th>beam</th>
<th>plate depth [%]</th>
<th>$E_w I_w + E_s I_s$[kNm²]</th>
<th>$EI$[kNm²]</th>
<th>$G_w$[GPa]</th>
<th>$kG$[GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>0</td>
<td>419</td>
<td>419</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>No.2</td>
<td>0</td>
<td>386</td>
<td>386</td>
<td>0.44</td>
<td>0.47</td>
</tr>
<tr>
<td>No.3</td>
<td>15</td>
<td>897</td>
<td>868</td>
<td>0.39</td>
<td>0.43</td>
</tr>
<tr>
<td>No.4</td>
<td>15</td>
<td>914</td>
<td>786</td>
<td>0.36</td>
<td>0.63</td>
</tr>
<tr>
<td>No.5</td>
<td>25</td>
<td>1042</td>
<td>975</td>
<td>0.54</td>
<td>0.53</td>
</tr>
<tr>
<td>No.6</td>
<td>25</td>
<td>1104</td>
<td>977</td>
<td>0.44</td>
<td>0.62</td>
</tr>
<tr>
<td>No.7</td>
<td>35</td>
<td>1203</td>
<td>1154</td>
<td>0.40</td>
<td>0.59</td>
</tr>
<tr>
<td>No.8</td>
<td>35</td>
<td>1210</td>
<td>1313</td>
<td>0.37</td>
<td>0.45</td>
</tr>
</tbody>
</table>
6. Conclusion

We estimated shear moduli or coefficients of steel-plate-inserted glulam beams by numerical, analytical and experimental approaches. Although numerically estimated shear modulus is getting larger along with the increase of the inserted plate depth, experimentally estimated shear modulus is not clearly getting larger along with the increase of the inserted plate depth, probably because of imperfections such as bonding conditions, slit cutting and so on. Numerical simulation for steel-plate-inserted glulam beams may be useful to estimate bending stiffness but not useful to estimate shear stiffness.

7. References

[1] ANALYSIS AND TESTS FOR LATERAL BUCKLING OF GLUED LAMINATED TIMBERS


