Design models and rational solutions of timber trusses

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Summary

The questions under discussion in this paper are devoted to assumptions to be employed for the general design considerations when girder type timber truss with mechanical fasteners (dowel-type and connectors) in joints is selected as load bearing structure. To gain deeper insight into design procedure some issues are discussed: 1) relationships between load attached and bearing capacity of main elements, and material consumption depending on connection type used and truss geometry selected; 2) suggestions for idealization of the actual timber structure in order to form the design model as accurately as it is practicable and leading to evaluation of the most unfavourable internal actions and maximal displacements taking into account the slips of elements in joints.

The study is aimed to elaboration of recommendations for design of timber trusses in order to provide the alternative procedure for preliminary design considerations and for checking of design results as well.

Key words: timber trusses, design.

1. Introduction

The timber trusses to carry loads over long spans may found a sufficient good acceptance as a roof system or as a floor structure at the proper conditions. During last decades the most efforts are devoted to elaboration of automatic design tools for nail plate structures, and high level of performance and cost-effectiveness has been accomplished in this field. At the same time timber trusses of different types of dowel type fasteners (nails, bolts, dowels) and connectors (toothed rings, split rings) have been remained „in shadow”. There are good possibilities for use of comprehensive computer aided design tools for calculations and drawing making in separate steps not for design procedure as a whole.

Behaviour of timber structure always deals with many factors simultaneously affecting each other and/or resulting in more or less reliable and effective solution. Designer of timber truss is advised to take proper account of all affecting factors of the system described in many text books (Ozelton, Baird, 1976; Faherty, Williamson, 1999) and analysed by researchers (Timber Engineering. Step 1, and Step 2, 1995; Šilih et al, 2005), and summarized there as follows:

1) truss profile and web system to be suitable consistent with the architectural requirements;
2) distances between panel points along chords and spacing between trusses corresponding to predicted loading conditions and sizes of prefabricate units of covering materials;
3) span/depth ratio being responsible for overall stiffness of system;
4) connection type in nodes and type, and sizes of mechanical fasteners to be selected appropriate to values of forces to be transmitted;
5) type and grade of materials in relationship with required load bearing capacity of elements and with design considerations for nodes,
6) possible design models to be analysed to find out the most unfavourable loading condition for nodes and elements leading to more reliable design results,
7) slip in joints including free displacement due to clearances in holes, and due to elastic, plastic and creep deformations produced under loads being distinctive in their duration,
8) other actual conditions and requirements set by customer.

Current study is based on the analysis of results of truss designs produced by using of software for structural analysis and CAD programs for detail design.

2. Consideration on timber truss parameters and fastener types

2.1 Span/depth ratio

According comprehensive point of view there is no span-depth ratio limit for trusses- their dimensions are controlled by the strength and stiffness conditions of the structure. As the height of truss is the main parameter necessary at the first step of design procedure the first assumption on required depth of girder truss covering the span $L$ may be found from the serviceability limit state condition for maximal deflection ($u_{fin}$) of simple supported beam:

$$u_{fin} = \frac{5 \cdot q_k \cdot L^4}{384 \cdot E_m \cdot J_{yo}} \leq \frac{L}{n_o}, \quad [1]$$

where $q_k$ is the total value of characteristic load,
$E_m$ is the mean value of modulus of elasticity chord material,
$n_o$ is the limiting parameter for deflection, $n_o=300...600$,
$J_{yo}$ is moment of inertia about stiff axis $y_o$ of truss section (Fig. 1):

$$J_{yo} = k_I \cdot 2 \cdot A_{ch} \cdot (H/2)^2, \quad [2]$$

where $k_I$ is the reduction factor for the moment of inertia considering the flexibility in the shear planes between chord and web members, it is assumed from different designs that $k_I \approx 0.2...0.5$;
$A_{ch}$ is the approximate value of the cross section found from the strength condition of the bottom chord in axial tension ($N_{t,0,d} A_{ch} \leq f_{t,0,d}$) and expressed as follows:

$$A_{ch} = \frac{q_k \cdot \gamma_f \cdot L^2}{8 \cdot H \cdot f_{t,0,d}}, \quad [3]$$

where $\gamma_f$ is partial factor between design and characteristic value of loads and $f_{t,0,d}$ is the design resistance of bottom chord material in tension.

Substituting equation [3] into equation [2] and after into [1] the desired depth $H$ may be found:

$$H = \frac{5 \cdot n_o \cdot f_{t,0,d} \cdot L}{12 \cdot E_m \cdot \gamma_f}. \quad [4]$$

Strength-stability condition for the upper chord section in the plane of truss is described using Eurocode 5 condition:

$$\frac{N_{c,d}}{A \cdot k_{c,y} \cdot f_{c,0,d}} + \frac{M_d}{W_y \cdot f_{m,d}} \leq 1. \quad [5]$$
Fig. 1 Design model for preliminary analysis of girder truss

It is considered that in the middle section of the girder truss the equilibrium is maintained when bending moment of chord force’s couple \( (N_{c,d} \cdot H) \) is equal to that produced by external loads \( (q_d L^2/8) \). Sequentially for parallel chord truss the axial force \( N_{c,d} \) in upper chord may be found from static equilibrium:

\[
N_{c,d} = q_d \cdot L^2 / 8 = N_{t,d} \cdot H,
\]

where \( q_d \) is uniformly distributed design load per unit length of upper chord. Bending moment caused by cross load applied between node points may be calculated approximately as for section at the first support of continuous beam: \( M_d = q_d \cdot d^2 / 10 \). Modulus of chord section area is \( W_y = b \cdot (h_{up})^2 / 6 \), where width of section may be adopted for \( b \approx L/100 \). After substituting values into equation [5] the required height of section for upper chord may be expressed as follows:

\[
h_{up} = \frac{q \cdot L^2 \cdot f_{m,d} + \sqrt{q^2 \cdot f_{m,d}^2 \cdot L^4 + 153.6 \cdot b \cdot q \cdot f_{m,d} \cdot k_{c,y}^2 \cdot H^2 \cdot d^2 \cdot f_{c,d}^2}}{16 \cdot H \cdot k_{c,y} \cdot b \cdot f_{m,d} \cdot f_{c,0,d}},
\]

where \( k_{c,y} \) is the buckling factor for upper chord element in the plane of truss, in first assumption may be adopted \( k_{c,y} = 0.8...0.9 \).

When joint solution is clear the new value of required section for bottom chord accordingly strength condition in tension may be calculated using equation:

\[
h_b = \frac{q_d \cdot L^2 / (8H) + f_{t,0,d} \cdot n_r \cdot d_b \cdot (b - n_t \cdot t)}{f_{t,0,d} \cdot (b - n_t \cdot t)},
\]

where \( n_r \) and \( n_t \) are the numbers of fastener rows within depth and number of steel plates within width of bottom chord, and \( t \) is the thickness of steel plate.

2.2 Solution of heavy loaded joints

It is known wood as organic cellular material exibiting higher strength properties in compression, bending and tension (in grain direction) than in shear and tension perpendicular to grain direction because of a large variety of connection types have been devised in order to transmit forces between members avoiding chrusching of wood. Wide range of mechanical fasteners are offered by manufacture, and these differ significantly as regard load bearing capacity values and slips produced during transmitting of shear forces between elements in joints. The solution of the support joint (Fig. 2) is the main dominant in the design procedure for pitched roof trusses.
Fig. 2 Design of joints: a- nailed joint, b- joint with toothed rings, c- joint with dowels and middle steel plate; $F_{V,Rd}$- load bearing capacity of fastener per shear plane (EC 5), $n_s$- number of shear planes, $nrd_{ef}$ and $nrbd_{ef}$- effective number of fasteners in row along grain direction

\[
\begin{align*}
a_{2b} &= a_{1d} \cdot \sin \alpha \quad \text{for } a_{2b} \geq a_2 \\
a_{1d} &= a_{2b} / \sin \alpha \quad \text{for } a_{1d} \geq a_1 \\
a_{2d} &= a_{1b} \cdot \sin \alpha \quad \text{for } a_{2d} \geq a_2 \\
a_{1b} &= a_{2d} / \sin \alpha \quad \text{for } a_{1b} \geq a_1
\end{align*}
\]

\[
N_{c,d} \leq F_{V,Rd} \cdot nrd_{ef} \cdot nrbd \cdot n_s
\]

\[
N_{t,d} \leq F_{V,Rd} \cdot nrbd_{ef} \cdot nrd \cdot n_s
\]

$t_p \geq 8d$

$t_f \geq 4d$
It is more difficult task for timber designer to pass the shortcoming in shear strength of wood when he should to choose optimal parameters and to place the fasteners around the joint area in more efficient way. In order to create some algorithm for analysis of joints with mechanical fasteners the following judgement is presented. Using Eurocode 5 conditions for minimal distances ($a_1$ and $a_4$) between center points of fasteners the geometric parameters for required joint area can be described by the following equations:

$$D = (n_{rd} - 1) \cdot a_{1d} + 2 \cdot a_4 / \sin \alpha,$$  \[5\]

$$B = (n_{rb} - 1) \cdot a_{1b} + 2 \cdot a_4 / \sin \alpha,$$  \[6\]

where $n_{rd}, n_{rb}$ - number of fasteners within row falled upon the grain direction of diagonal and bottom chord member correspondingly,

$a_{1d}, a_{1b}$ - distances between fasteners’ center points in the grain direction of diagonal member and bottom chord correspondingly,

$a_4$ - distance from the center point of fastener to edge of timber element across the grain.

Required depths of bottom chord and diagonal member sections ($h_b$ and $h_d$ correspondingly) from conditions for displacement of fasteners we have:

$$h_b = D \cdot \sin \alpha$$  \[7\] and $$h_d = C \cdot \sin \alpha.$$  \[8\]

Which type of fastener to be chosen? There is some more significant factors to be considered as regard expected stiffness and economy of structure: joint slip, wood volume and steel mass per unit of transferred shear force (see Table 1).

For illustration the relationships between increasing rates of fastener diameter and load bearing capacity, material consumption and slips in joints are presented in Figure 3. It is clear that using the toothed rings in connection we can achieve more effective employment of wood volume in joint. There is observed more rapid increase of required wood volume in comparison with load bearing capacity by increasing of nail diameter.

![Fig. 3 Rates of increasing of joint parameters: a- joints with toothed rings, b- nailed joints](image-url)
Table 1. Characteristics of joints with mechanical fasteners

<table>
<thead>
<tr>
<th>Type of connection</th>
<th>Connection and fastener parameters</th>
<th>Predicted slip per fastener’s capacity, mm</th>
<th>Material consumption per unit of transferred shear force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nailed connection without pre-drilling of holes</td>
<td>$t/d = 7; d = 3.4$</td>
<td>1.2</td>
<td>45...55, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t/d = 7; d = 4.2$</td>
<td>1.5</td>
<td>60...80, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t/d = 7; d = 5.5$</td>
<td>1.9</td>
<td>110...140, steel, g/kN</td>
</tr>
<tr>
<td>Dowel-type (diameter $d$, mm) double shear connection</td>
<td>$t_1/d = 4; d = 12$</td>
<td>1.2</td>
<td>36, steel, g/kN</td>
</tr>
<tr>
<td>with central steel plate ($t = 8$ mm) and timber side members; thickness $t_1$, mm</td>
<td>$t_1/d = 5; d = 12$</td>
<td>1.3</td>
<td>38, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 6; d = 12$</td>
<td>1.5</td>
<td>42, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 7; d = 12$</td>
<td>1.7</td>
<td>42, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 8; d = 12$</td>
<td>1.8</td>
<td>47, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 6; d = 8$</td>
<td>1.1</td>
<td>27, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 6; d = 10$</td>
<td>1.3</td>
<td>33, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$t_1/d = 6; d = 16$</td>
<td>1.9</td>
<td>58, steel, g/kN</td>
</tr>
<tr>
<td>Toothed rings (type C1 according EN 912, $d_c$ ring diameter, mm)</td>
<td>$d_c = 48$</td>
<td>0.68</td>
<td>46, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 62$</td>
<td>0.77</td>
<td>70, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 75$</td>
<td>0.84</td>
<td>85, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 95$</td>
<td>0.95</td>
<td>106, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 117$</td>
<td>1.06</td>
<td>148, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 140$</td>
<td>1.16</td>
<td>162, steel, g/kN</td>
</tr>
<tr>
<td></td>
<td>$d_c = 165$</td>
<td>1.25</td>
<td>176, steel, g/kN</td>
</tr>
</tbody>
</table>

3. Consideration on truss analysis

3.1 Design models

It is relatively simple matter to apply comprehensive design software to the known structural model. It is not possible to achieve complete similarity between assumed design model and real timber structure however we should near completion, and in this regard some items are pointed out.

The mechanical fasteners at the splices of chords inherent rotational stiffness in some degree are stressed by additional forces caused by bending moment (Fig. 4). It is observed that more often the splitting occur in tensile chord and at the ends of elements as well.

Neglecting of joint stiffness effects during design of structure may be factor leading to hard in-service problems the most frequently occurring. Considering existing uncertainties in rotational stiffness of joints the conservative approach should be maintained- to perform all really possible design models alternately fixing and releasing the joints in order to find the unfavourable combinations and values of internal forces and displacements (Fig. 5).
Fig. 5 Design models of girder truss and diagrams of internal forces: a- combined system with straight bottom chord, b- combined system considering camber of bottom chord made in different points along panel, c- the same system assuming rotationally stiff joints, d- more unfavourable design model considering slips in joints
Not to describe the design procedure of girder trusses in details there is pointed out the main steps:

1) determination of span/depth ratio for truss, panel length, planning of web member geometry preferring profile with compressed main diagonal (at support node), not tensioned as it is observed often in comprehensive construction practice;

2) calculation of internal forces using appropriate design model and member stiffness characteristics according approximately determined section sizes;

3) output of detailed truss drawing;

4) compilation of more adequate design model in order to check for maximal values of internal forces and more unfavourable loading situations for heavy loaded joints;

5) verification the strength and stability conditions for members and load bearing capacity of joints

6) verification of stiffness of system taking into account predicted slip in joints and creep of timber materials used (Timber Engineering, 1995; Holzbau-Taschenbuch, 1991).

4. Conclusions

1. All possible design models for large span trusses should be treated including the ones with cambered bottom chord and the most unfavourable internal forces must be considered and maximal displacements predicted.

2. Absolute values of slips in joints occurred under action of transferred forces when their values are equal to connection capacity must be limited by codes in order to avoid quite large flexibility and to ensure the functional ability of fasteners for whole service life.

5. References


