Summary

This article provides a first approach of the problem of characterizing badly known traditional timber joints and frameworks: the first section deals with calculating bending stiffnesses of barefaced tenon joints, and the second section applies the results to the study of the framings of the O-L Cathedral in Tournai (Belgium). To characterize the behaviour of the framings, different types of joints and geometries have been studied. In order to validate the results, we have compared analytical (component method), numerical (3D finite elements models) and experimental approaches. In order to study real framings of the O-L Cathedral in Tournai, the effective stiffness of the joints were introduced in an EC5 diagnostic of the structure. Such pathologies as for example missing dowels, reduced sections and faulty pieces were also taken into account. Results obtained were compared to diagnosis results of a "ideal" framework.

1. Introduction

In the field of restoration of buildings, engineers have to work with old structures made of well or badly preserved timber elements. Timber frameworks are one of the most important and widespread patrimonial structures. Their configurations and joints are most of the time complex and testify to a high levelled craft and an excellent understanding of frameworks behaviour. Today, the aim is here to understand all specific problems to propose efficient and consistent solutions to guarantee the stability of the framings for the future.

Unfortunately, these structures are vulnerable, present generally a high level of hyperstaticity and damaged joints can lead to a modification of the global behaviour of the structure. Therefore, it is fundamental to integrate timber joints stiffnesses into a framework study.

The cathedral of Tournai in Belgium has been recognized in 2000 by UNESCO as part of the World Heritage. Composed by a slender gothic choir, a robust roman nave an a "transition style" transept surmounted by 5 towers rising at 80 meters high, the cathedral of Tournai (see Fig. 1 and 2) is a unique evidence of different architectural styles through the ages. In this work, we will first study the bending stiffness of barefaced shouldered tenon joints, considering theoretical, numerical and experimental methods. Then, we will use these results to study the behaviour of a roof frame/truss of the Tournai's O-L Cathedral Choir carpentry.
2. Timber joint stiffness

2.1 Component method

The first method used to obtain rotational stiffness is the component method. The latter gives stiffness results for joints according to geometrical and mecanical properties. Frequently used in steel construction, it has already been used in traditional timber joints studies by [M. Drdácký et.al].

Fig. 1 shows a barefaced tenon joint, submitted to a bending moment. Compression zones appear, which are the only ones to contribute to the global stiffness of the joint. It is supposed that the dowel resists to shear force and fixes the position of the center of rotation of the connection. No friction is taken into account. Because of assembly conditions, there is no contact at the bottom of the mortise.

The stiffness of each zone in compression is defined by:

\[ k_c = \frac{F_c}{\delta_c} \quad [1] \]

The deformation \( \delta_c \) can be obtained by means of Lambert and Whitman’s solution, on basis of theoretical deformation of an elastic half space:

\[ \delta_c = \frac{F_c \times 0.85}{E_a \times \sqrt{A}} \quad [2] \]

where \( F_c \) is the compression force, \( A \) the area of compressed surface and \( E_a \) the modulus of elasticity according to the direction \( \alpha \) of the grain.

The stiffness can be expressed by:

\[ k_c = \frac{F_c}{\delta_c} = \frac{E_a \times \sqrt{L \times a}}{0.85} \quad [3] \]

We can associate a stiffness to each couple of surfaces in contact:

\[ k_{i,j} = \frac{1}{\sum_j \frac{1}{k_{i,j}}} \quad [4] \]

As the moment can be expressed:

\[ M = \sum_i F_i \times Z_i = \sum_i (k_i \times \delta_i) \times Z_i \quad [5] \]

And as we consider minor movements:

\[ \delta_i = Z_i \times \sin \varphi = Z_i \times \varphi \quad [6] \]

We can express the global stiffness of the timber joint:

\[ M = k \times \varphi \]

\[ \Rightarrow k = \frac{M}{\varphi} = \sum_i k_i \times Z_i^2 \quad [N.m/ mrad] \]
As we consider the joint in its integrality (3-D), we obtain stiffness results for positive and negative applied moments and for different joint configurations. These will be compared to the finite element analysis and tests results. This method can be easily computed, and would give quick results for a large range of joints. Nevertheless, it would be interesting to deepen it, in order to quantify the errors coming from the taken hypothesis.

2.2 Finite element analysis

The model is composed of two parts (Fig. 2). They are put in contact by means of 89 non linear springs, working only in compression. We take into account that the dowel resists to the shear force by a fixed node in the tenon piece. The mortice piece is completely held on its base to avoid any rotation. The material orthotropic elastic properties are obtained from tests on cubes. For loading, a pure bending moment is applied on the top of the vertical piece.

Other simulations were carried out to deduce the influence of some parameters, as for instance isotropy, springs stiffness, dowel position, “surface-to-surface” contact. Here are some conclusions:

- orthotropic material properties must be taken in account,
- spring stiffness have no major influence on the stiffness of the joint,
- the asset of “surface-to-surface” contact was the possibility of introducing friction parameters. Unfortunately these models take much more CPU ressources.

2.3 Tests

In old carpentry joints, for tied connections, timber dowels were generally used. For these connections, special shear tests of the dowel were made to validate the assumptions (Fig. 3).

In order to validate the results obtained by means of the component method and the numerical simulations, a test was performed on a replica of a historical barefaced shouldered tenon joint. It was made from Douglas pinewood.
Material properties were determined by tests on coupons taken from the joint assemblies. The evaluated material properties were the elastic modulus in compression, parallel and perpendicular to the grain. These results were also used for finite elements models and for component method, so that we could compare with the test results.

The horizontal part (with the mortise) is held tight to the rack (Fig. 4). A pure bending moment is applied to the vertical piece. Counterweights and the self weight of the device used to load the joint cancel each other. We measured vertical displacements of two points distant from 60 cm, so we can plot the bending stiffness/applied moment curve below for each load (Fig.5), and compare it to theoretical and numerical results. The joint was tested until collapse. It failed by excess of shearing in the tenon.

The average stiffness calculated from the test results is 43,6 N.m/mrad. Component method result was 39,3 N.m/mrad. Numerical simulation result was 32,9 N.m/mrad. Here are some conclusions:

• the analytic method provides good results, according to the test. Nevertheless, it would be very interesting to deepen the theory, which could provide quick and handy results,
• it's possible to simulate contact between two bodies with simple numerical models by means of non linear trusses. Results obtained so are acceptable,
• it seems knowledgeable to work out models considering friction, in order to quantify its effect on the result.

The dowel resisted to the shear force. The joint failed by excess of shearing in the tenon (Fig.6). Breaking load was a bending moment of 72 540 N.cm. If we consider a mechanism of failure, we obtain a breaking moment of 77 200 N.cm, which is a result we can consider as acceptable, according to the test.

At this time, some experimentations are still in progress at the Polytechnic Faculty of Mons to validate the numerical study.
3. Timber framework study

Fig.7 Different geometry studied (30, 45 and 60 degrees)

In general, basic assumptions are made for the study of old timber structures. Joints are supposed to be moment free or fully restrained. Of course, the real behaviour of the structure is between these two simplified model. In fact, the joints are partially restrained. Because of important sections used in old timber structures, in many case only the serviceability limit states are restrictive. In this case, taking into account the rotational stiffness of the joints in a global model of the frame can be useful.

Framings of the gothic choir of the O-l Cathedral of Tournai are made of large section of timber (until 70x30cm). The large tie-beam support the stone vault in some places and control the lateral thrust of the roof. At 4 and 7 meters above the tie-beam, two collars support the rafters (Fig.8).

So that we could understand the influence of joints stiffness on the framework behaviour, the analytical and numerical methods were applied to several geometries of joint (Fig.7). They represent almost all the type of joints we can find in a typical frame the Cathedral choir (Fig.8). Material properties were determined by compression tests on ancient oak, obtaining thus the modulus of elasticity in compression, parallel and perpendicular to the fibres.

Results of bending stiffness obtained by the component method and the numerical models were similar. They are shown at the table I for positive applied moments.

<table>
<thead>
<tr>
<th>Joint</th>
<th>Component method</th>
<th>Numerical simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 degrees</td>
<td>498</td>
<td>769</td>
</tr>
<tr>
<td>45 degrees</td>
<td>1 311</td>
<td>1 770</td>
</tr>
<tr>
<td>60 degrees</td>
<td>2 432</td>
<td>2 046</td>
</tr>
</tbody>
</table>

Table I
Bending stiffness for tenon joints under positive applied bending moment [N.m/mrad]

Effective stiffness of each joint was introduced in a Eurocode 5 automatic diagnostic. In order to quantify their influence on the behaviour if the structure, we compared a fully hinged frame with a fully restrained one, as we can see at the Fig.9.
As expected, the introduction of the real stiffness of the joints has an important influence on the behaviour of the structure. In this case, the introduction of this parameter in the analysis led to solve a stability problem in one of the rafters (Fig.9). Such pathologies as missing dowels, reduced sections and faulty pieces were also taken in account, and the results compared to results obtained from a “ideal” framework.

4. Conclusions

Historic timber joints are complex, and even if they are used for centuries, it is still arduous to lay down numbers on their behaviour.

The component method gives quick and handy results, quite similar to the test results, and can be easily computed. However, it would be interesting to deepen the theory, in order to quantify precisely the error coming from the taken hypothesis.

It is possible, using quiet simple models, to properly simulate the contact between two orthotropic bodies, by means of non linear trusses. Results obtained seem acceptable, considering test results. It would be interesting to introduce contacts and friction in the numerical models, in order to ripen the analysis. It would lead to an optimization of the process, and allow to deal quickly with a large range of traditional and non traditional timber joints.

Considering stiffnesses when diagnosing frameworks leads to a modification of the structure behaviour. So it would be possible to predict the behaviour of the structure when accounting patchinesses, such as deficient or absent joints, reduced sections. It would lead to a better solution, as regards of restoration, considering ethical, technical and economic criterias.

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6. References


