Summary

In addition to the stress calculation the stability analysis of slender laminated timber beams is of importance, because slender beams under bending may fail due to lateral torsional buckling. The assessment of the lateral torsional buckling depends among others on the imperfections of the system; increasing imperfections decrease the safety against stability failure. In contrast to short-term behavior, stability behavior concerned by changing and varied stiffness across section and length due to long-term effects has not yet been considered; creep and non-uniform swelling or shrinkage do not only increase the vertical but also the horizontal deflections. To obtain the influences of creep and shrinkage, the verification of lateral torsional buckling is extended by a rheological model of timber.

Keywords: Lateral torsional buckling, stability, time-dependent behavior, creep

1. Lateral torsional buckling, State of the art

According to the current German and European standards DIN 1052 [4], Eurocode 5 [5] safety against lateral torsional buckling may be verified in two ways: On one hand by given equations following the equivalent column buckling method. For this purpose, the tilting-factors \( k_m \) and if a compressive force is available, the column buckling factors \( k_c \) are determined. The present compressive and bending stresses are set in relation to the compression and bending strength, which are reduced by the factors \( k_c \) and \( k_m \). On the other hand the standards allow the stress calculation and verification according to second-order-theory under consideration of imperfections.

By regarding for example a single span girder with torsional supports at the end, a span of \( L \) and double-symmetric cross section (Fig. 1(a)) under constant loading moment \( M_{y,d} \), disregarding the vertical deflections and taking into account an initial sinusoidal curvature corresponding to a maximum eccentricity \( v_0 \), the differential equations can be solved. The equations for deformation and rotation are (see also Table 1 for terms):

\[
v(x) = \frac{M^2_{y,d} \cdot v_0}{M^2_{k_i,d} - M^2_{y,d}} \cdot \sin \left( \frac{\pi \cdot x}{L} \right) \tag{1}
\]

\[
\vartheta(x) = \frac{(E I_z)_d \cdot \frac{\pi^2}{L^2} \cdot M_{y,d} \cdot v_0}{M^2_{k_i,d} - M^2_{y,d}} \cdot \sin \left( \frac{\pi \cdot x}{L} \right) \tag{2}
\]

Thereby it is possible to determine the internal forces according to second-order-theory (see Fig. 1(b)) at the middle of the beam:

\[
M^I_{y,d} = \gamma_F \cdot M_{y,k} \tag{3}
\]

\[
M^I_{z,d} = (E I_z)_d \cdot v'' = E_d I_z \cdot \left[ -M^2_{y,d} \cdot \frac{v_0}{-M^2_{k_i,d} + M^2_{y,d}} \right] \cdot \frac{\pi^2}{L^2} = \vartheta \cdot M_{y,d} \tag{4}
\]
with the ideal elastic lateral torsional buckling moment
\[ M_{ki,d} = \frac{\pi}{L} \cdot \sqrt{E_d I_z \cdot G_d I_T} \] (5)

Based on the equation (3) and (4), it is possible to calculate the stresses according to second-order-theory, see equation (6).
\[ \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_{red} \cdot \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \quad \text{and} \quad k_{red} \cdot \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \] (6)

with
\[ \sigma_{m,y,d} = \frac{M_{y,d}}{W_y} \quad \text{and} \quad \sigma_{m,z,d} = \frac{M_{z,d}}{W_z} \] (7)

According to Eq. (3) and Eq. (4) larger imperfections lead to an increase of the internal forces. With the assumption of an utilization of 100 % for the check (Eq. (6)) a maximum bending moment \( M_{y,d} \) can be determined. In Fig. 2 the ratio of \( (M_{y,d}/W_y)/f_{m,y,d} \) = utilization \( \sigma_y \) is shown for different cross sections over varied span lengths.

The diagram in Fig. 2 shows that the degree of utilization of the main-bending stress (utilization \( \sigma_y \)) is smaller for higher imperfections: with increasing imperfections the maximum allowable moment \( M_{y,d} \) decreases.

The imperfections may result from fabrication tolerances but also be due to creep deformations or non-uniform shrinking deformations.

**Fig. 2: Influence of imperfection**

**2. Effects of long-term behavior on lateral torsional buckling**

In the currently valid German standard (DIN 1052 [4]) the influence of the long-term behavior is considered on one hand by the modification factor \( k_{mod} \) on the side of the strength, because timber
under load in the course of the time exhibits a substantial loss of its strength. On the other hand, there is the factor $k_{def}$, with which creeping deformations may be determined. Since the long-term behavior depends substantially on the wood moisture, these factors are indicated in dependence on the climatic conditions.

The influence of long-term behavior has only to be considered for the stability check at ultimate limit state, if the design value of the permanent load of structural members under compression in service class 2 and 3 is higher than 70% of the total load. Then, the stiffness is reduced by a factor $1/(1+k_{def})$ regardless whether the equivalent column method or a calculation according to the second-order-theory is applied.

For the stability verification according to second-order-theory, creeping can also be taken into account by an additional imperfection, which is proportional to the elastic deformation. However it has to be noticed, that there is a relation between applied load and creep deformation: The applied load at a time $t$ causes elastic deformations as well as creep deformations. This leads again to higher load stresses and so to higher elastic deformations and additional creep deformations. Considering creep as an additional imperfection, a new approach is developed as follows, without reducing the rigidities:

- Determination of the elastic deformation $v_{el}$ in dependence on the imperfection $v_0$ in the middle of the beam

$$v_{el}(t = \infty) = \frac{M^2_{y,d}}{M^2_{ki,d} - M^2_{y,d}} \cdot v_0(t = \infty) \quad (8)$$

- Calculation of the imperfection $v_0$ existing at the time $t = \infty$, which consists of the initial imperfection value $v_0$ at $t = 0$ and the creep deformation up to the time $t = \infty$.

$$v_0(t = \infty) = v_0(t = 0) + k_{def} \cdot v_{el}(t = \infty) \quad (9)$$

- Thus one receives a safe-sided relation for the internal forces in dependence on the creep factor $k_{def}$.

$$M_{y,d}^{II} = \gamma_F \cdot M_{y,k} \quad M_{z,d}^{II} = E_d I_z \cdot \left[ -M^2_{y,d} \cdot \frac{v_0}{-M^2_{ki,d} + M^2_{y,d} + M^2_{y,d} \cdot k_{def}} \right] \frac{\pi^2}{L^2} \quad (10)$$

The diagram in Fig. 3 shows the differences between the utilizations of bending stresses $\sigma_y$ for a stability check according to second-order-theory with an horizontal imperfection as given in the standards ($L/400$) considering creeping by reduced rigidities and on the other side by the new approach, where creep is considered as additional imperfection. This is shown for different cross sections and for varying span length $L$.

The utilizations determined with the new approach lie above the approaches of the standards, so higher bending moments compared to the standards [4], [5] may be applied. By analysing long-term behavior more detailed the design of beams becomes more efficient.

Moreover, the actual analysis method for stability of slender laminated timber beams do not allow the consideration of long-term behavior due to
changing moisture, although the reducing influence of constant moisture regarding ultimate load is known, see [2], [9].

As shown also in Fig. 2, there is an influence from existing imperfections concerning the carrying capacity of beams. The size of applied imperfections can be determined on the basis of standards [4], [5]. The creep deformations are then determined by using deformation factor $k_{def}$. This factor is given in dependence on the climatic conditions. When comparing several rheological models like [1], [6] and [11], it is noticed that the creep factors $k_{def}$ of the standards are disregarding particularly the influence of changing moisture, which even for class 2 climatic condition causes clearly higher creep factors. If there is a change of moisture along the time, the mechano-sorptive effect appears, i.e. the creep deformations under simultaneous mechanical use and moisture sorption become clearly higher than under constant load and constant moisture. For a relative humidity between 35% and 90% (see Fig. 4(a)), corresponding roughly to wood moisture of class 2, the creep factors according to the model of Hanhijärvi [6] are determined. In Fig. 4(b) it is obvious, that already after 5 months (nearly 3500 h) as a result of changing moistures the creep factors (relationship of actual strain to elastic strain) are clearly higher compared with the creep factors under constant climatic conditions.

Fig. 4: Influence of different moistening to the creep behavior [6]

beam increases the imperfections. This non-uniform moisture distribution can e.g. occur when for a beam the humidity can penetrate on one side faster and deeper into the glued laminated timber beam than on the other protected side. The beam swells more on the moist side compared to the less moist side.

So e.g., if a glued laminated timber beam has a different humidity content at the top than in the lower part, the longitudinal deformations due to the change of moisture (shrinking/swelling) should not be neglected. Due to the different strains on the upper and lower surface of the beam a curvature results and a substantial vertical deflection of the beam occurs.

A possible built-in situation for such a construction is e.g. a roof girder, which is embedded into the isolation and is exposed in the winter at the top side to the unheated attic and at the lower surface to the heated interior climate. As another example an edge beam supported by columns may be affected on one side by the external and on the other side by the interior climate. By the climatic differences it comes to a non-linear moisture distribution in the cross section, which leads to strains, from which horizontal deformations result and where the influences on the load-carrying capacity of such a girder are to be examined.

If e.g. a beam exhibiting an initial moisture of 14 % and a constant environment climate of 18 °C and 30% of relative humidity is humidified from three sides, a strong curvature will arise. The wood moisture along the cross section can be determined at the points 0 to 10 in dependence on the increasing time $t$ for two different beams, see Fig. 5.

The non-linear moisture content along the cross section leads to an additional curvature resulting in an imperfection which should not be neglected, see Fig. 6. As shown in Fig. 6(b), there is a high curvature at the beginning as a result of three-sided moistening which diminishes with the course of
time. Regarding a slender cross section (18/162), the curvatures are clearly higher than for a more compact (18/18) cross section, see Fig. 6(b). In order to study the influence of shrinking of the arising maximum values of curvature an equivalent imperfection is determined which can be compared to the imperfections given in the standard, see Fig. 7.

The values of the standard of $L/400$, e.g. 5 cm maximum deformation for a length of $L = 20$ m, is exceeded for a cross section 18/18 by approx. 1.8-times and for a beam of 18/162 even by 2.8-times, see Fig. 7. This large eccentricity due to three-sided humidification increases the effect of lateral torsional buckling and thereby reduces the load carrying capacity of the beam.

For slender laminated timber beams, e.g. in hall constructions, where mostly the stability check is decisive, so far the long-term behavior is not yet considered. The neglect of the time-dependent strain can however lead to safety deficiencies, because by creeping not only the vertical deformations but also the horizontal deformations are increased. Thus, by these larger deformations the stresses due to second-order-theory are increased, so that under consideration of creeping smaller load-carrying capacities may be expected. This has also been observed by [9] and [2]. Therefore, the effects of the long-term behavior under changing climates have to be analysed more precisely in order to become more calculable.
3. Calculation of long-term behavior

3.1 Numerical simulation

The influence of creeping as well as swelling and shrinking of wood has to be analysed during alternating climatic conditions to study the lateral torsional buckling safety of slender laminated timber beams computationally. On the basis of a program named *kriho* [8], which was developed at the Institute for Structural Design, University of Stuttgart, for the determining parameter range - varying cross sections, girder lengths, loading time, climates - parameter studies should be accomplished.

With the help of this program *kriho* the strains and/or curvatures are computed related to the axis. From there the strains in the cross section are calculated in width and height. A scheme of the program sequence is shown in Fig. 8.

![Fig. 8: Procedure of kriho](image)

The program *kriho* needs as input values:

- geometry of construction unit and conditions of support
- material parameters (with consideration of distribution)
- load duration, height of load, kind of load (load history)
- climate conditions (distribution of humidity/temperature).

In order to acquire the long-term behavior computationally, different rheological models ([6], [11]) are used. These models give data regarding the size of the change of strains over the time due to creeping and shrinking/swelling with consideration of climatic conditions and load history.

From these the deformations as well as the stresses may be determined for the time step regarded in each case. For one period e.g. of 50 years the influences of changing climatic conditions or loads may be evaluated and calculated by specified time increments. Thereby it is possible to determine the influences of creeping, creeping due to a change of moisture (mechano-sorptive creep), shrinking and swelling. The procedures presented were developed in [8] where the long-term behavior of composite slab of board stack and concrete was determined and verified for one-sided moisture demand, by experiments and field measurements.

As the influence of the moisture conditions, e.g. moisture changes, highly affects the long-term behavior, based on second-order-theory the differential equations for a beam with consideration of the time-dependent behavior are set up and solved. The determination of the long-term behavior is hereby carried out with *kriho*. The stability problem treated here is solved with the help of the equilibrium method. Based on geometrical relations, material laws and equilibrium conditions the determining differential equations are formulated as equilibrium of the deformed system (Second-Order-Theory). Generally, the following relations apply ([7], [3]):

\[
0 = EI_y \cdot \dddot{w}_{el} - F \cdot \dddot{w}_{ges} + (M_z \cdot \dddot{\vartheta}_{ges}) - q_y \\
0 = EI_z \cdot \dddot{v}_{el} - F \cdot \dddot{v}_{ges} \cdot (z_s - z_M) + (M_{yz} \cdot \dddot{\vartheta}_{ges}) - q_y \\
0 = GI_T \cdot \dddot{\vartheta}_{el} - EI_w \cdot \dddot{v}_{el} - F \cdot \dddot{v}_{ges} \cdot (z_s - z_M) - M_{yz} \cdot \dddot{\vartheta}_{ges} - M_z \cdot \dddot{w}_{ges} \\
+ F \cdot \vartheta_{ges} \cdot (i^2 + z_M^2 + z_a \cdot r_z - 2 \cdot z_a \cdot z_M) - q_y(\dddot{\vartheta}_{el} \cdot e_y + e_z) - q_z(\dddot{\vartheta}_{el} \cdot e_z - e_y)
\]
These three basic equations are extended by a term which considers the long-term behavior of the timber. Here, it is assumed that the total deformations (Index \( \text{tot} \)) consist of an elastic portion (Index \( \text{el} \)) and a portion from imperfection (Index \( \text{0} \)) and/or creep deformations (Index \( \text{kri} \)). Thus it can be written as follows:

\[
\begin{align*}
    w_{\text{tot}} & = w_{\text{el}} + w_{\text{0}} + w_{\text{kri}} \\
    v_{\text{tot}} & = v_{\text{el}} + v_{\text{0}} + v_{\text{kri}} \\
    \vartheta_{\text{tot}} & = \vartheta_{\text{el}} + \vartheta_{\text{0}} + \vartheta_{\text{kri}}
\end{align*}
\]

These equations are implemented in a computer program. As input values for the calculations the imperfections are taken from the standard [4]. With this calculation procedure the overall curvatures, the creep curvatures, as well as the stiffness of the cross section with respect to the current moisture content are determined for every time step (comp. [10]), using the rheological model according to [11].

First results of calculations are shown in Fig. 9: For a single span girder with constant bending moment \( M_{y,d} \) and horizontal imperfection \( (L/400) \) under constant climatic conditions the internal moment \( M_{z,d}^{II} \) according second-order-theory are determined for different cross sections. If long-term behavior is not considered, the moments \( M_{z,d}^{II} \) are constant with the time. By considering long-term behavior the moments \( M_{z,d} \) will increase whereas the increase depends of slenderness of cross section, see Fig. 9. These internal forces are then the base for stability check according second-order-theory.

**Fig. 9: First results of calculation**
4. Outlook

On the basis of parameter studies the decisive parameters are to be identified and be built into an analytic model [10]. The complex calculation model has to be simplified for an application in practice on the safe side. So the long-term behavior might be considered e.g. by modified tilting factors and/or modified material indices or e.g. by reduced cross sections. By the evaluation of the simulation results a more exact and at the same time simpler analysis of the carrying and deformation behavior will be possible for the design of slender laminated timber beams. The aim is to develop a model that allows a safe and economical design of all variables.

References


