Modeling moisture exposure on timber structures

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Abstract
This paper presents a developed moisture exposure methodology based on metrological data by using time series analysis. It was found that temperature and vapour concentration could be modeled by a combination of deterministic functions and stochastic processes, which consider the dynamics of the climate. Specifically, a model for temperature and vapour concentration in Stockholm, Sweden, is presented together with simulation results.

Keywords: timber, moisture, stochastic process, time series modeling

1. Introduction
The load bearing capacity of structural timber elements is affected by the surrounding climate. One critical factor is the moisture content in the ambient air. Varying relative humidity, and thus moisture content, induces internal stresses perpendicular to grain. These stresses may cause cracks which reduce the load bearing capacity of the individual timber element and thus the whole construction. In reality, tension perpendicular to grain is probably the most common failure mode [1]. Although load-bearing capacity may be of most concern, cracks also involve aesthetical considerations, affecting the competitiveness of timber as a construction material.

Today, different climatic conditions are accounted for by assigning timber structures to a service class. This approach, however, does not take into account the rate of moisture variation, or the nature of such variations. Instead, the selection of service class is only based on anticipated equilibrium moisture levels. In order to incorporate more rational and scientifically based design methods (reflecting the nature of timber exposed to moisture), the induced internal stresses may be regarded as an ordinary design action to be combined with effects from other loads. For this purpose, a moisture exposure model that reflects the nature of the variations—the dynamics—of moisture in the ambient air is desired. Since the temperature is needed for conversion of outdoor moisture levels to corresponding indoor levels, the model should also include temperature. In addition, the diffusion rate in timber is temperature dependent. This paper describes a general and possible approach to establish such a moisture model. Specifically, the model is applied on data for climate conditions in Stockholm, Sweden.

2. Modeling

2.1 General
In order to establish a methodology, time series representing instant values of temperature, T, and relative humidity, RH, at geographically different locations in Sweden were obtained from the Swedish Meteorological and Hydrological Institute (SMHI). The data record for Stockholm encompassed approximately 40 consecutive years starting in the beginning of the 60s with observation intervals between 3 and 12 hours. The raw data was transformed to time series of daily
averages of $T$ and RH. One reason for this is that the hygroscopic response of timber is highly time dependent and thus fast changes (e.g. hourly) of surface conditions will only have significant effect on very thin wood elements, e.g. panels [2]. Another reason is that a time series should have a constant sampling interval (which the observation series did not have) to be readily analyzed.

Modeling of moisture exposure involves relative humidity, vapour concentration and temperature. Since the vapour concentration was not given in the time series, it must be calculated from Eq. (1),

$$v = \varphi v_{\text{sat}}(T), \quad \text{Eq. (1)}$$

where $v$ is the vapour concentration, $\varphi$ is the relative humidity and $v_{\text{sat}} = v_{\text{sat}}(T)$ is the vapour concentration at saturation point. $v_{\text{sat}}$ changes with temperature according to a physically determined relation. The three above-mentioned quantities, i.e., temperature, relative humidity and vapour concentration, are stochastic processes with random patterns as illustrated for Stockholm from 1981 to 1984 in Fig. 1 (left plots) below. A simple model could be obtained by randomly picking e.g. vapour concentrations and temperatures from the measured empirical distributions. This approach, however, does not take into account the persistence of the climate, or the correlation between the stochastic variables. Instead, by using time series analysis, it is possible to model the characteristics of the recorded observations in a more adequate way. One general and frequently used process is the linear ARMA($p,q$) process, where ARMA stands for auto regressive moving average. The process can also be extended with an external part, as shown in Eq. (2), in order to consider influence from other quantities. It is then called an ARMAX($p,r,q$) process.

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \sum_{i=0}^{q} b_i u(t-i) + \sum_{i=1}^{r} c_i e(t-i) + e(t), \quad \text{Eq. (2)}$$

where $y(t)$ is the modeled quantity e.g. temperature, $u(t)$ is the external input and $e(t)$ represents the innovations which are assumed to be white noise and uncorrelated with past values $y(t-1), y(t-2), y(t-3)$... etc; $t$ is discrete and represents days; $a_i$, $b_i$ and $c_i$ are constants [3,4].

The observed time series are apt to be auto-regressive. For example, a warm day is more likely to be followed by a warm one than a cold one—there is a kind of inertia against changes. This is easily verified by calculating correlation coefficients for the observations at different time intervals, i.e. days. Hence, the use of an auto-regressive model is realistic. Since $T$, $v$ and $\varphi$ are deterministically related, see Eq. (1), it is only necessary to model two of the three stochastic variables. Which ones to choose depend on their random properties. It is clear that none of the variables is stationary since all three series reveal distinct seasonal patterns. Because an ARMA process is weakly stationary, i.e. the average $m(t)$ is constant and the covariance function $r(s,t)$ only depend on the time difference $(t-s)$, the series have to be decomposed. The used decomposition is described by Eq. (3).

$$y(t) = F\{y^o(t)\} - d(t), \quad \text{Eq. (3)}$$

where $y(t)$ is the obtained stationary process, $y^o(t)$ is the original time series, $d(t)$ is a deterministic function describing seasonal variations, and $F$ is a suitable transformation function considering varying variance.

### 2.2 Decomposition

By visual inspection, the temperature and the vapour concentration seem to be the best candidates to model; the relative humidity displays changes that are more irregular. Moreover, in air with constant vapour concentration, the relative humidity varies with temperature and thus one might suspect that modeling will be more complicated; changes in temperature have a more direct influence on relative humidity than on vapour concentration. The decomposition was done in two steps. First, by making range-mean plots for intervals of 30 days for temperature and vapour concentration (shown in Fig. 1, right plots), it was found that the variance is fairly constant for temperature, but increasing for vapour concentration (the range represents the difference of max and min during a period). This problem, however, was circumvented through a Box-Cox transformation, see Eq. (4), where $F$ is used to indicate the connection to Eq. (3). This
transformation makes it possible to stabilize the variance and $\lambda$ set to 0.45 gave a good result. Secondly, the seasonal variations were removed by fitting a periodic function described by Eq. (5). The assumed result of the transformation is a zero-mean, stationary time series.

\[ F(y^o) = ((y^o)^{\lambda} - 1)/\lambda; \quad \lambda \neq 0, \quad \text{Eq. (4)} \]
\[ d(t) = C_0 + C_1 \cos(2\pi t / \tau) + C_2 \sin(2\pi t / \tau), \quad \text{Eq. (5)} \]

where $C_0$, $C_1$ and $C_2$ are constants and the period $\tau = 365.25$ days (considering leap year).

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**Fig. 1** A sample of the time series from Stockholm (left plots) and range-mean plots for the temperature and the vapour concentration (right plots).
2.3 Estimation of stochastic process

After the decomposition, the parameters of the transformed stochastic processes were estimated using a prediction error method [3, section 10.2], minimizing the sum of squared one-step ahead prediction error, i.e. the residuals. It was however not apparent which model order to use. One frequently used information criterion to facilitate the identification procedure is Akaike’s Final Prediction Error, which rewards a reduced sum of squared residuals. Because this criterion may not provide the optimal order estimate, pole-zero plots of the transfer functions were used in order to avoid model structures with unnecessary high orders, or with large uncertainties of the parameter estimates. Moreover, the cross-correlation function for $y_T$ and $y_v$ (indices are self-explanatory) showed that they were correlated. Hence, it was not possible to use two ARMA models being independent of each other. However, it was found that by modeling the temperature with an ARMA model, the vapour concentration could be modeled by an ARMAX model.

2.4 Model validation

After the estimation, the two models were validated by checking the residuals and their ability to reproduce new data. If the estimated model is close enough to the physical behavior of the process, the process $\{e(t)\}$ will be white noise. This means that the residuals are to be uncorrelated in time, i.e., the auto correlation function, ACF, for the residuals should be significantly equal to zero at all lags but lag zero. Furthermore, an integrated periodogram was used to visualize how much the residuals diverted from white noise in the frequency domain. Finally, to verify the reproduction of data, different statistical quantities, such as the periodogram (spectrum) and the auto-correlation function, were determined and compared with the results for the empirical data.

3. Result and Discussion

After transforming the observed temperature time series by removal of the seasonal variations, the remaining zero-mean series

$$y_T(t) = y_T^o(t) - 7.228 + 9.855 \cos(2\pi t / 365.25) + 3.633 \sin(2\pi t / 365.25)$$

could be modeled by an ARMA(4,1) process. Actually, the FPE information criterion suggested ARMA(6,3) but the pole-zero plot revealed that the confidence intervals of two pairs of poles and zeros overlapped and thus the order was reduced. The results of some of the validation tests are displayed in Fig. 2a-b. It was shown that the residuals were significantly close to white noise. In addition, a cross correlation function of the residuals and the past outputs $y_T$ demonstrated that they were uncorrelated as assumed. However, after plotting histograms over the residuals for each of the months of the year, it turned out that their distribution varied over the year. Consequently, in order not to lose model accuracy, the temperature innovations, when used in simulations of the moisture exposure on the timber structure, were to be bootstrapped [5] with replacement from the twelve empirical distributions representing the monthly residuals.

Because the range-mean plot for the vapour concentration showed a noticeable trend, a Box-Cox transformation was performed before the seasonal variations were removed in analogy to the temperature. The equivalent time series was thus expressed as

$$y_v(t) = ((y_v^o(t))^{0.45} - 1)/0.45 - 2.679 + 1.037 \cos(2\pi t / 365.25) + 0.618 \sin(2\pi t / 365.25)$$

and found to be reasonably well modeled by an ARMAX(4,2,2) process with the temperature $y_T$ as the external input. Although the variation of the vapour concentration can be described by a stochastic process, there is a physical upper bound that must be considered. The vapour concentration can never be higher than the concentration at saturation point. In order to include this deterministic limit in the model, the vapour concentration innovations, which also were bootstrapped from monthly distributions for the same reason as were the temperature ones, were not allowed to produce physically impossible results. Consequently, when the simulations of the vapour concentrations gave $v > v_s$, $e_v(t)$ was assigned a value giving $v = v_s$. Fig. 2c-d presents some validations tests indicating a good model structure, whereas Fig. 2e-f presents a comparison of modeled and simulated temperature and vapour concentration. In summary, the established system for $y_T$ and $y_v$ is illustrated in Fig. 3 together with the estimated parameters. For evaluation purpose, the same model structure (but with different parameter estimates) was used to make a model for
Luleå (in northern Sweden, by the northern part of the Gulf of Bothnia, same record period as for Stockholm) and Sturup (in southern Sweden, close to Malmö, record period 1972-95). It turned out better than expected, especially for Sturup. Nevertheless, some problems could not be disregarded; for example, for Luleå, the simulations at low temperature were not accurate; the model produced far too low values for the vapour concentration. This may partly be explained by the temperature time series, having a larger range during winter than summer (in contrast to Stockholm). Hence, it is motivated to use different model structures for different climatic locations. It may also be motivated to use different models for different periods in time, e.g. summer and winter, depending on the application. Note that the model may be used for other purposes than modeling moisture exposure on timber structures.

![Fig. 2 Results from validation test for the temperature model (a-b) and vapour concentration (c-d), and a comparison of observed and simulated values (e-f); dashed lines represent a 95% confidence interval.](image-url)
Finally, as a comparison between Stockholm and Sturup, “characteristic” values for minimum as well as maximum temperature, relative humidity and moisture content in wood are listed in Table 1 (defined as 2 and 98 percentiles of annual minimum and maximum, respectively). The results are based on a 1000-year simulation sequence. It is interesting to note the significant difference in RH and moisture content between the two locations.

Table 1. Comparison between Sturup and Stockholm. Min and max represent values occurring in average every 50 years. Moisture content is based on assumed equilibrium with daily average RH.

<table>
<thead>
<tr>
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<th>Sturup</th>
<th>Stockholm</th>
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<tbody>
<tr>
<td><strong>Temperature [°C]</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>-15.2</td>
<td>-18.9</td>
</tr>
<tr>
<td>max</td>
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<td>32.8</td>
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<tr>
<td><strong>Relative humidity [%]</strong></td>
<td></td>
<td></td>
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<tr>
<td>min</td>
<td>32.5</td>
<td>19.3</td>
</tr>
<tr>
<td>max</td>
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<td>100.0</td>
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<tr>
<td><strong>Moisture content [%]</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Swedish spruce)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>8.3</td>
<td>5.8</td>
</tr>
<tr>
<td>max</td>
<td>31.6</td>
<td>31.6</td>
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References

Fig. 3.3 Stochastic system describing $y_T$ and $y_v$, where the residuals $e_T$ and $e_v$ are bootstrapped from the monthly empirical distributions obtained as part of the modeling. 

$$e_T(t) \rightarrow y_T(t) = 1.796 y_T(t-1) - 0.9399 y_T(t-2) + 0.1837 y_T(t-3) - 0.04795 y_T(t-4) - 0.9319 e_T(t-1) + e_T(t)$$

$$e_v(t) \rightarrow y_v(t) = 0.038 y_v(t-1) - 0.09924 y_v(t-3) - 0.03722 y_v(t-3) + 0.01846 y_v(t-4) + 0.1028 y_T(t) - 0.936 y_T(t-1) - 0.9319 e_v(t-1) - 0.1275 e_v(t-2) + e_v(t)$$