Splitting strength of beams loaded perpendicular to grain by connections, a fracture mechanical approach

Summary
In this paper a model is presented able to describe the splitting strength of beams subjected to tension forces perpendicular to the grain by connections. Based on fracture mechanic principles the model is basically independent of the type of fastener but validated here for connections with dowel-type fasteners. Taking test data from a limited number of sources the model is calibrated. The influence of the connection type is demonstrated and a code design proposal for Eurocode 5 is presented.

Keywords: Timber, splitting, connections, fracture mechanics.

1. Introduction
It is well known that the perpendicular to grain strength of structural timber is small compared to the axial strength. However, in the design of timber structures it is hard to avoid any forces that make an angle to the grain direction as for instance by fasteners or connectors in truss nodes. The prediction of splitting cracks has always been a problem for researchers and particular for designers as structural timber design codes were very vague about this phenomenon. In an attempt to solve this problem empirical models were developed as in a number of design codes. However, these empirical methods have a limited applicability, as the data evaluated range does not cover all possible cases. The development and application of fracture mechanics gave way to new to tackle this problem. For splitting of beams with notches a fracture mechanical model was derived by Gustafsson [2] and Van der Put [3]. Van der Put applied his model also
for splitting caused by connections. The latter is more clearly explained in Van der Put et al. [4]. At first only the experimental results of Ehlbeck et al. [1] were used for calibration purposes. This paper includes experimental results from other sources to validate the fracture mechanical model. In contrast to other publications the focus is on the influence of the connection type and the splitting capacity.

2. Splitting failure

When connection members are not in line there will always be load components perpendicular to the grain that may cause splits even before the connection itself has failed. Splits caused by connections have much resemblance with the splits of notched beams, Figure 1. In both cases unstable crack growth, which will mainly propagate in grain direction cause unexpected failure.

Starting point of the model is a simply supported single span beam loaded by a connection at mid span. In Figure 2 the static scheme is given for one symmetrical part of the partially cracked state of the beam. The cracked beam is subjected to a total load of $2V$. Obviously, the schematisation is a simplification of physical reality. Not only will the cracks propagate in grain direction as shown in Figure 2 in the initial stage cracks may also originate from the shear plane of the connections and propagate in a perpendicular to the grain direction particularly when slender dowel type fasteners are used. The derivation of the fracture mechanical model in [4] indicates that shear deformation (Mode II) is the dominating mechanism and not the tensile stresses perpendicular to the grain (Mode I) as many empirical models assume.

The derivation of the fracture mechanical model is given in [4] and a simplified version reads.

$$V_f = \frac{\sqrt{GG_c h}}{b h} \sqrt{0.6(1-\alpha)}$$

where $\alpha = \frac{h_c}{h}$

and:

- $V_f$ is the maximum shear force on either side of the connection,
- $b$ are the timber member thickness,
- $h$ is the timber member depth,
- $h_c$ is the loaded edge distance to the centre of the most distant fastener,
- $G$ is the shear modulus,
- $G_c$ is the apparent fracture energy release rate,

In equation (1) the only unknown is the apparent fracture mechanical parameter $\sqrt{GG_c}$. It is envisaged that the crack opening mode is always a combination of fracture mode I and II as mentioned earlier. The value
of this parameter depends on the specific conditions under which crack opening or unstable crack growth takes place, which among others depend on the fastener type. In this respect it should be mentioned not to mix the apparent fracture parameter with the fracture parameter $G_f$ of mode I, which is used by other models and is derived from special tests with standardised test pieces, Larsen and Gustafson [5]. At this stage it is virtually impossible to estimate the influence of the failure mode of the fastener in relation to the apparent fracture parameter as test data taken from literature were not tailored to check this aspects. For this reason a lower bound approach is taken to derive the apparent value of the apparent fracture parameter, $vGG_f$ by evaluation of test data. It is obvious that besides splitting for relative slender beams with span to depth ratio of about 5 to 7 the governing failure mode will in many cases still be bending.

3. The influence of the connection strength

In this paper test results published by only four researchers are used to validate and calibrate the model, Ehlbeck et al.[1], Ballerini [6], Reske et al. [7] and Reffolds et al. [8] as the number of pages for this contribution is very limited. The majority of the tests concerned connections with dowel type fasteners. For understanding the behaviour of the splitting phenomenon the behaviour of the connections should also be taken into account. For this reason the following classification is made.

- Type A – The connection is much stronger than the splitting strength of the beam and therefore called over-designed. It can be identified by relative low embedment stresses at splitting failure or by its nearly straight load-slip behaviour. This type will trigger high value of the apparent fracture parameter as crack initiation stresses are developing over the whole thickness, Figure 3.

- Type B – In this case the connection strength equals the splitting strength, which is considered as the optimal design solution. The embedment stresses are high.

- Type C – The connection shows high embedment stresses and yielding is followed by hardening. This connection still is able to force splitting failure after considerable slip although splitting is not the primary failure mode. This is an under-designed connection. The value for the apparent fracture parameter will be low.

- Type D – This connection is under designed and splitting will not occur.

4. Model validation and influence of connection failure mode

4.1 Connections with nails and dowels

The problem with the evaluation of the available test data is that neither of them aimed at verifying a certain physical model and for this reason it is not always easy to assign the connections to Type A, B or C of Fig.3. In most cases neither the load-slip curves are not reported nor the slip at splitting failure.

The test data evaluated first deals with dowel type fastener connections. It is well known that the failure mode of dowel type fasteners is dependent among other parameters of the slenderness ratio. Connections with stocky or rigid fasteners can be assigned to any of the Types given above. Plasticity occurs when the embedment stress capacity is exhausted afterwards plastic deformation occurs by a plastic movement of the stocky fastener that cuts through the cross-section.

Fig 4 Crack growth when slender dowel type fasteners are used
Slender fasteners development plastic hinges as shown in Figure 4. In the latter case cracks will gradually develop and grow along and perpendicular to grain. Crack growth is different in both cases and so is the apparent fracture parameter.

The test data by Ehlbeck et al. [1] consists of connections with a number of fastener types. The majority of the tests consist of a freely supported single span beams loaded at mid span by a connection. A few tests were on cantilevered beams with a connection near the end of the cantilever. In Figures 5 and 6 only some of the data is presented for connections with 16 mm diameter dowels and 3.8 mm diameter nails. The beam dimensions ranged from 40x180 mm$^2$ for the tests with nails up to 100x1200 mm$^2$ for the tests with dowels. The solid curves in Figure 5 and 6 show the ability to fit the data with equation (1). Evaluation of all series showed apparent fracture parameter values of 20 N/mm$^{1.5}$ and as low as 12 N/mm$^{1.5}$, which indicate Type A and C connections.

Some tests reported by Ballerini [6] are now evaluated. The span of the beams was 3400 mm and the beam dimensions were 40x196 and 40x397 mm. The span to height ratio is considerable 17.3 for the smallest beams and 8.6 for the largest beams. The connection was made with one and two 10 mm diameter dowels in line with the force. The dowels fitted in thick metal plates that didn’t allow any dowel rotation. These dowels can be considered are rigid. The slip of the connections was measured. The author reports that except for the connections where the dowels were very close to the loaded edge the mode of failure was more or less plastic. In [6] two load-slip diagrams are presented that show plastic deformations of 4 and 12 mm. In Figure 10 the model fit is presented based on the mean apparent fracture parameter $\sqrt{G_{e}} \approx 12.7$ N/mm$^{1.5}$. This value agrees well with the one obtained in Ehlbecks tests for Type C connections.

### 4.2 Connections with bolts and steelplates

Similar tests as Ehlbeck et al. [1] were carried out and reported by Reshke et al. [7] using bolts. In addition to the free supported single span beams loaded in the middle Reshke et al. reports cantilevered beams loaded at the free end. The steelplates used as side members in the steel-timber-steel connection were of 9.5 mm thickness and fastened with 19 and 12.7 mm diameter bolts. The glued laminated beams were 130x190 mm and 80x190 mm cross-section. Besides the
maximum load also the slip of the connection at failure was reported. The slip values measured ranged from 0.9 mm for connections with 6 bolts to 20 mm for connections with 1 bolt. Having fitted equation (1) to Reshke’s experimental data the apparent fracture parameter indeed relates to the number of bolts and therefore to the connection Type, Figure 7. There appears to be a critical number of bolts of about 3 beyond which the apparent fracture parameter doesn’t increase, $\sqrt{G_G} = 34 \text{ N/mm}^{1.5}$ as earlier explained by Van der Put et al. [3]. It seems so that applying more bolts the behaviour changes from Type C for one bolt to Type A for 6 bolts. The tightening of the bolt after the connection assembly might cause the high value of 34 N/mm$^{1.5}$ for this Type A connection. The mean slip of the single bolt connections is 8 mm resulting in a mean apparent fracture parameter of $\sqrt{G_G} = 14 \text{ N/mm}^{1.5}$. Still the failure mode of the single bolt connection was not reported and therefore it is uncertain whether this is the lower bound value or that even lower values can be expected for very slender fasteners.

Yasumura [8] also reported a limited number of experiments of steel-to-timber connections with 16 mm bolts and 12 mm thick steel plates. The glued laminated beam depth varies from 224 mm up to 392 mm all having a thickness of 64 mm. The load-slip curves reported clearly show yielding and hardening prior to splitting typical for Type C. After yielding at 40% to 60% of the maximum load considerable hardening occurred. Finally after 10 to 20 mm slip, the timber beam failed by splitting. As the fasteners did not yield but cut through the cross-section these are typical Type C connections, where the connection failed first. These test results can be considered as being just over the edge what the model tries to capture. Nevertheless, fitting the model results in an apparent fracture parameter as low as 12.1 N/mm$^{1.5}$.

4.3 Connections with punched metal plates

To check and expand applicability of the model for other type of fasteners data by Reffolds et al. [9] was also evaluated. He used punched metal plate fasteners. The span of the beams was fixed to 600 mm while the beams tested were of two dimensions 35x145 mm and 45x145 mm. If anchorage failure or premature bending failure occurred the test data was omitted. This means that Type D connections were disregarded and the remaining assumed to belong to Type A, which needs verification. To check for connection length effects the punched metal plate dimension along the grain direction of the beam was increased from 63, 120, 200 to 401 mm. Because tests with 401 mm long plates in a span of 600 mm deviate considerably from the model assumptions of a point load it’s the author’s opinion that these results can better be omitted for model validation. Figure 9 shows the results. The top solid line that runs through the mean of the data is the model prediction with a fracture parameter of $\sqrt{G_G} \approx 20.1 \text{ N/mm}^{1.5}$. It can be concluded that the model is able to follow the data well. Furthermore, the apparent fracture parameter is close to that of Ehlbeck [1] tests for Type A connections.
5. Evaluation and conclusion

The model given by equation (1) is well able to describe the experimental results for connections with dowel type fasteners. Evaluation of the data shows as expected that the Type of connection, according to Figure 3, affect the value of the apparent fracture parameter significantly. Taking the mean lower bound of the apparent fracture parameter $\sqrt{G_G_c} = 12 \text{ N/mm}^{1.5}$ as starting point for a structural design code proposal it follows with equation (1) $\sqrt{(G_G_c / 0.6)} = 15.5 \text{ N/mm}^{1.5}$. In order to derive at a characteristic lower 5% value for the apparent fracture parameter it is further reduced to $15.5 \times 2/3 = 10.3 \text{ N/mm}^{1.5}$ so that finally the design formula reads.

$$V_u = 10b \left( \frac{h_c}{(1 - \frac{h_c}{h})} \right)^{1/2}$$

(2)

Where $V_u$ is the maximum design shear force on either side of the connection. As no hardwoods test have been evaluated (2) applies only for softwoods. Equation (2) could be proposed for timber design guidelines.

6. References